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Investment Foundations

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Module 3: INPUTS AND TOOLS

Chapter 8: Quantitative Concepts

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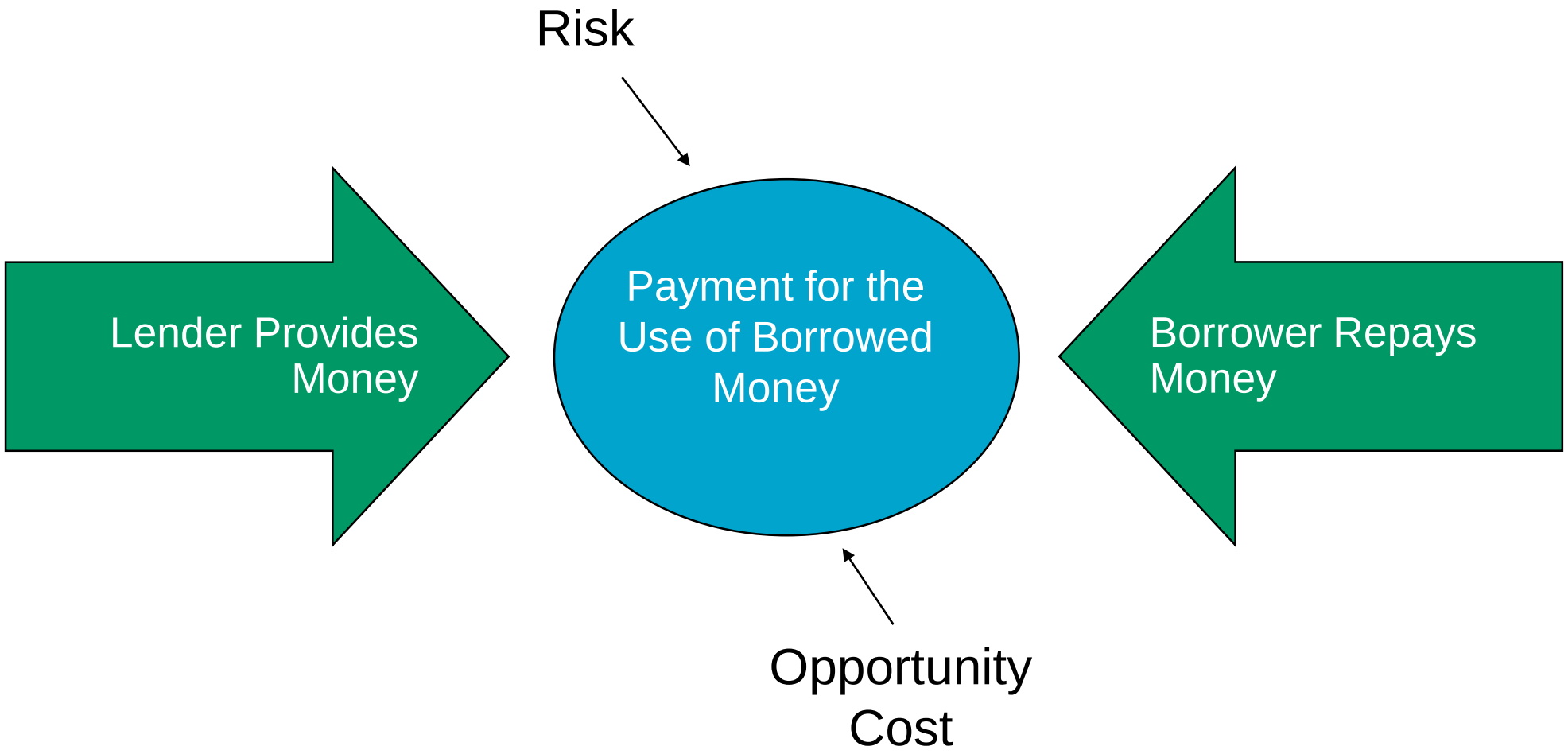
Faculty Bio

Module	Topic	Weight	LOS	Exam Qs	Hours to Study	Module Practice Qs	Chapter Practice Qs
Module 1	**Industry overview**	5%	7	6	5	28	28
Chapter 1	The Investment Industry: A Top-Down View						
Module 2	**Ethics and regulation**	10%	14	12	10	91	
Chapter 2	Ethics and Investment Professionalism						49
Chapter 3	Regulation						42
Module 3	**Inputs and tools**	20%	50	24	20	291	
Chapter 4	Microeconomics						53
Chapter 5	Macroeconomics						57
Chapter 6	Economics of International Trade						47
Chapter 7	Financial Statements						70
Chapter 8	Quantitative Concepts						64
Module 4	**Investment instruments**	20%	29	24	20	213	
Chapter 9	Debt Securities						69
Chapter 10	Equity Securities						72
Chapter 11	Derivatives						42
Chapter 12	Alternative Investments						30
Module 5	**Industry structure**	20%	27	24	20	96	
Chapter 13	Structure of the Investment Industry						28
Chapter 14	Investment Vehicles						29
Chapter 15	The Functioning of Financial Markets						39
Module 6	**Serving client needs**	5%	12	6	5	76	
Chapter 16	Investors and Their Needs						35
Chapter 17	Investment Management						41
Module 7	**Industry controls**	20%	24	24	20	154	
Chapter 18	Risk Management						51
Chapter 19	Performance Evaluation						53
Chapter 20	Investment Industry Documentation						50
	Total	**100%**	**163**	**120**	**100**	**949**	**949**

AFTER COMPLETING THIS CHAPTER, YOU SHOULD BE ABLE TO DO THE FOLLOWING:

a) Define the concept of interest;
b) Compare simple and compound interest;
c) Define present value, future value, and discount rate;
d) Describe how time and discount rate affect present and future values;
e) Explain the relevance of net present value in valuing financial investments;
f) Describe applications of time value of money;
g) Explain uses of mean, median, and mode, which are measures of frequency or central tendency;
h) Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion;
i) Describe and interpret the characteristics of a normal distribution;
j) Describe and interpret correlation.

INTEREST



LOS a: Define the concept of interest.

COMPOUNDING EFFECTS

Interest Earned on a £100 Deposit in Two Years Using Simple Interest

Year	Beginning Balance (£)	Interest Earned (£)	Ending Balance (£)
1	100	10	110
2	100	10	120
Total		20	

Interest Earned on a £100 Deposit in Two Years Using Compound Interest

Year	Beginning Balance (£)	Interest Earned (£)	Ending Balance (£)
1	100	10	110
2	110	11	121
Total		21	

LOS b: Compare simple and compound interest.

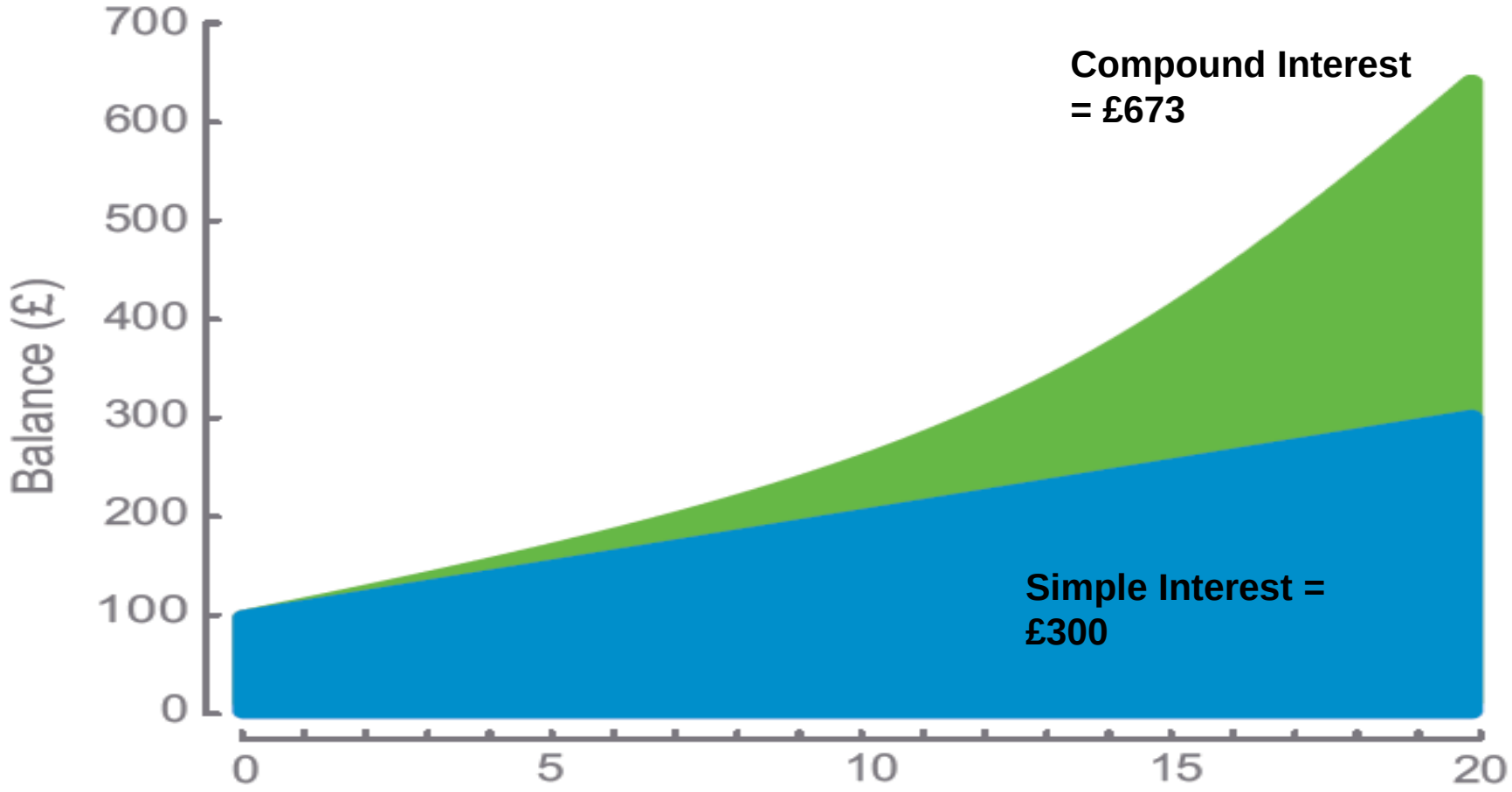
SIMPLE AND COMPOUND INTEREST

Assume £100, 10% per year, for two years
Simple interest:
Future value: = Original principal × (1 + (Simple interest rate × N)) = £100 × (1 + (0.10 × 2)) = £100 × (1.20) = £120
Compound interest
Earn interest on interest Future value: = Original principal × $(1 + \text{Simple interest rate})^{N}$ = £100 × $(1 + 0.10)^2$ = £100 × $(1.10)^2$ = £121

N = Number of periods

LOS b: Compare simple and compound interest.

EFFECTS ON SAVINGS OF SIMPLE AND COMPOUND INTEREST



LOS b: Compare simple and compound interest.

PRACTICE Q: EXPERT

Two banks each pay the same annual interest rate on their deposits. One bank pays simple interest, whereas the other pays the same rate compounded annually. If the same amount of principal is invested in each bank at the start of the year, at the end of the first year, the bank balances will be:

A. the same in both banks.

B. highest in the simple interest bank.

C. highest in the compound interest bank.

PRACTICE Q: EXPERT

Two banks each pay the same annual interest rate on their deposits. One bank pays simple interest, whereas the other pays the same rate compounded annually. If the same amount of principal is invested in each bank at the start of the year, **at the end of the first year**, the bank balances will be:

A. the same in both banks.

B. highest in the simple interest bank.

C. highest in the compound interest bank.

Interest Earned on a £100 Deposit in Two Years Using Simple Interest

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Total		20	

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Year	Beginning Balance (£)	Interest Earned (£)	Ending Balance (£)
1	100	10	110
2	110	11	121
Total		21	

A is correct. Interest earns interest in the compound interest bank, whereas it does not in the simple interest bank.

During the first year, however, there is only principal in both banks. At the end of the year, therefore, both bank balances will be the same: the principal plus the same amount of interest from both banks.

In subsequent years, the balance in the compound interest bank will be higher because at the start of each year after the first, that account will earn interest on both the principal and accumulated interest.

ANNUAL PERCENTAGE RATE (APR) AND EFFECTIVE ANNUAL RATE (EAR)

$$\text{EAR} = \left(1 + \frac{\text{APR}}{\text{NpY}}\right)^{\text{NpY}} - 1$$

NpY = Number of periods per year

	Simple Interest Rate or APR	Compound Interest Rate or EAR
Credit card (daily)	15.24%	$16.46\% = \left[\left(1 + \frac{0.1524}{365}\right)^{365}\right] - 1$
Bank deposit (monthly)	2.4% (= 0.2% × 12)	$2.43\% = \left[\left(1 + \frac{0.024}{12}\right)^{12}\right] - 1$
Loan (quarterly)	6.0%	$6.14\% = \left[\left(1 + \frac{0.06}{4}\right)^{4}\right] - 1$

LOS b: Compare simple and compound interest.

PRACTICE Q: EXPERT

The annual percentage rate (APR) is a(n):

A. simple interest rate.
B. compound interest rate.
C. annualised rate when paid more frequently than annually.

PRACTICE Q: EXPERT

The annual percentage rate (APR) is a(n):

A. simple interest rate.

B. compound interest rate.

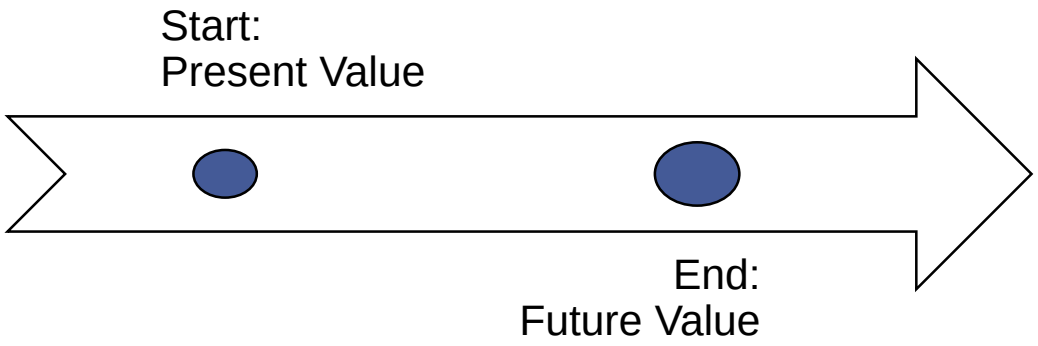
C. annualised rate when paid more frequently than annually.

A is correct. The APR is a simple interest rate that does not involve compounding.

	Simple Interest Rate or APR	Compound Interest Rate or EAR
Credit card (daily)	15.24%	$16.46\% = \left[\left(1 + \frac{0.1524}{365}\right)^{365}\right] - 1$
Bank deposit (monthly)	2.4% (= 0.2% × 12)	$2.43\% = \left[\left(1 + \frac{0.024}{12}\right)^{12}\right] - 1$
Loan (quarterly)	6.0%	$6.14\% = \left[\left(1 + \frac{0.06}{4}\right)^{4}\right] - 1$

FUTURE VALUE AND PRESENT VALUE

FV = Future value **PV = Present value**	***r* = Discount rate** **NoP = Number of periods**

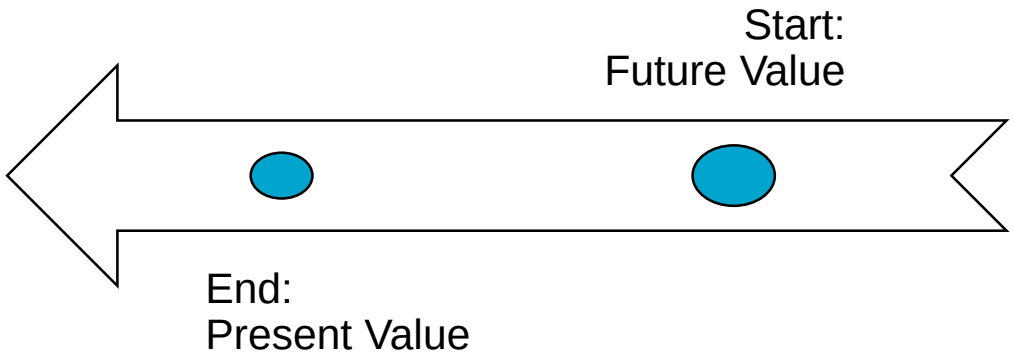


$$FV = PV \times (1 + r)^{NoP}$$

$$FV = 100 \times (1 + 0.10)^2 = 121.00$$

$$PV = \frac{FV}{(1+r)^{NoP}}$$

$$PV = \frac{121.00}{(1+0.10)^2} = 100.00$$



LOS c: Define present value, future value, and discount rate.

PRACTICE Q: EXPERT

The amount of money that is needed today to accumulate a stated amount in three years at a given interest rate is defined as the:

A. future value.
B. present value.
C. net present value.

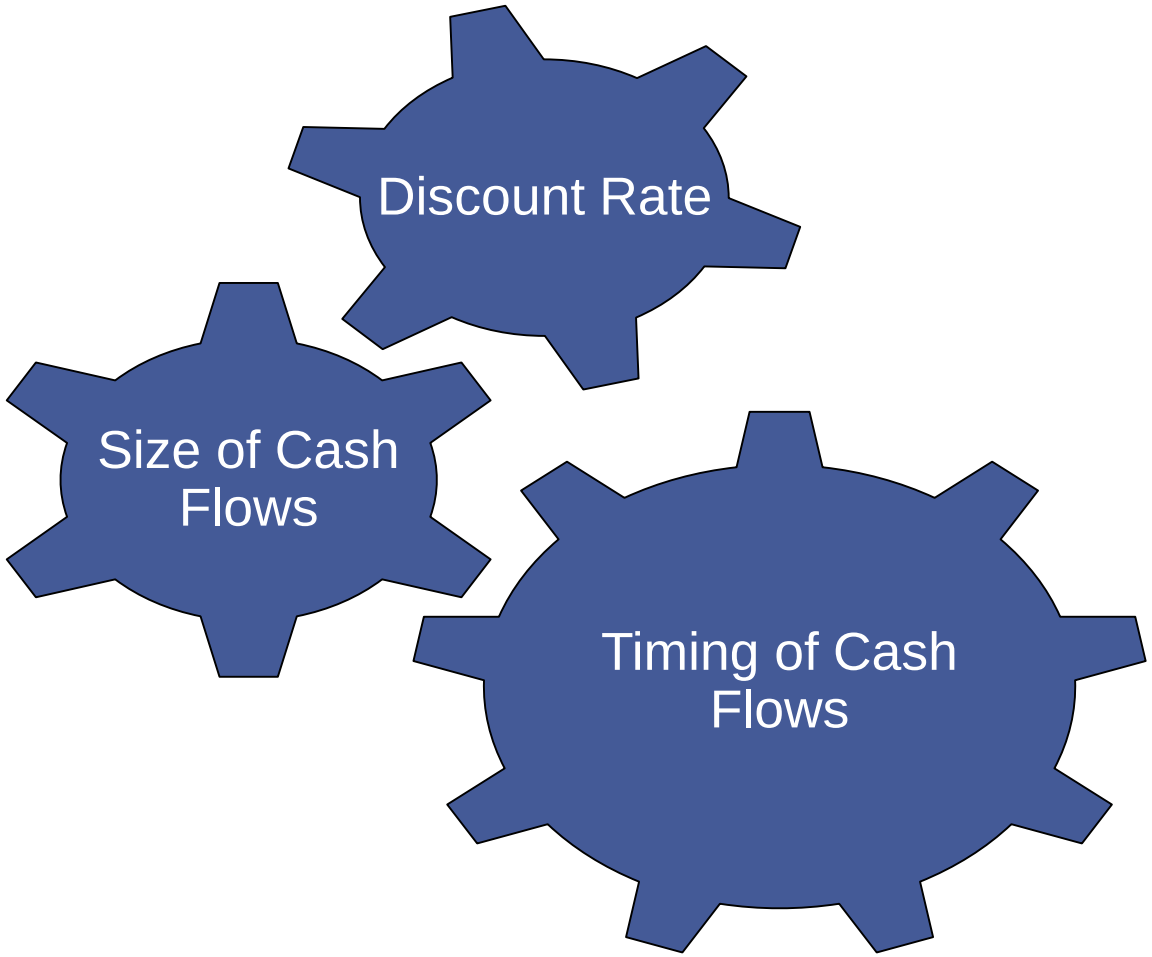
PRACTICE Q: EXPERT

The amount of money that is needed today to accumulate a stated amount in three years at a given interest rate is defined as the:

A. future value.
B. present value.
C. net present value.

B is correct. The present value is the amount of cash today that is equivalent in value to a stated amount to be received at a given time in the future when this future amount is discounted at the stated discount rate.

COMPARING INVESTMENTS



LOS d: Describe how time and discount rate affect present and future values.

EFFECTS OF TIME AND DISCOUNT RATE ON VALUE

Investment 1: Initial investment = £500
Present value of £1,000 in three years discounted at 9%
= £1,000 / $(1 + 0.09)^3$ = £772.18

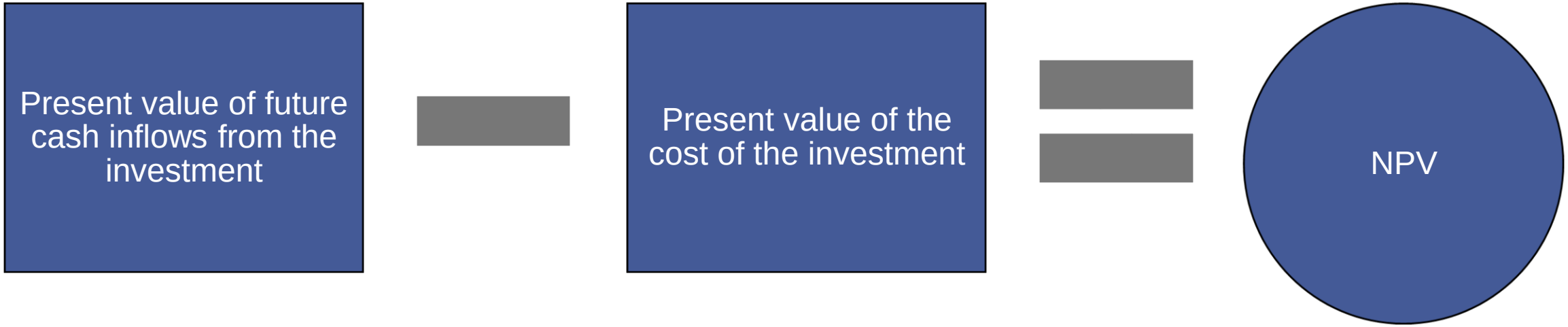
Investment 2: Initial investment = £500
Present value of £1,350 in five years discounted at 9%
= £1,350 / $(1 + 0.09)^5$ = £877.41

Investment 2 following a reassessment of risk:
Present value of £1,350 in five years discounted at 15%
= £1,350 / $(1 + 0.15)^5$ = £671.19

Note: If the initial investment is the same, you can compare present value of cash flows. Also, when the interest rate for Investment 2 increases, present value decreases.

LOS d: Describe how time and discount rate affect present and future values.

NET PRESENT VALUE (NPV)



Investment 1: Present value of £1,000 in three years, discounted at 9%, and cost £700.*

£1,000 / $(1.09)^3$ – £700 = £772.18 – £700.00 = £72.18

Investment 2: Present value of £1,350 in five years, discounted at 15%, and cost £500.

£1,350 / $(1.15)^5$ – £500 = £671.19 – £500.00 = £171.19

**Change in cost assumption from previous slide.*

LOS e: Explain the relevance of the net present value in valuing financial investments.

PRESENT VALUE AND THE VALUATION OF FINANCIAL INSTRUMENTS

8% interest rate, two-year loan for £100;
want 10% return.

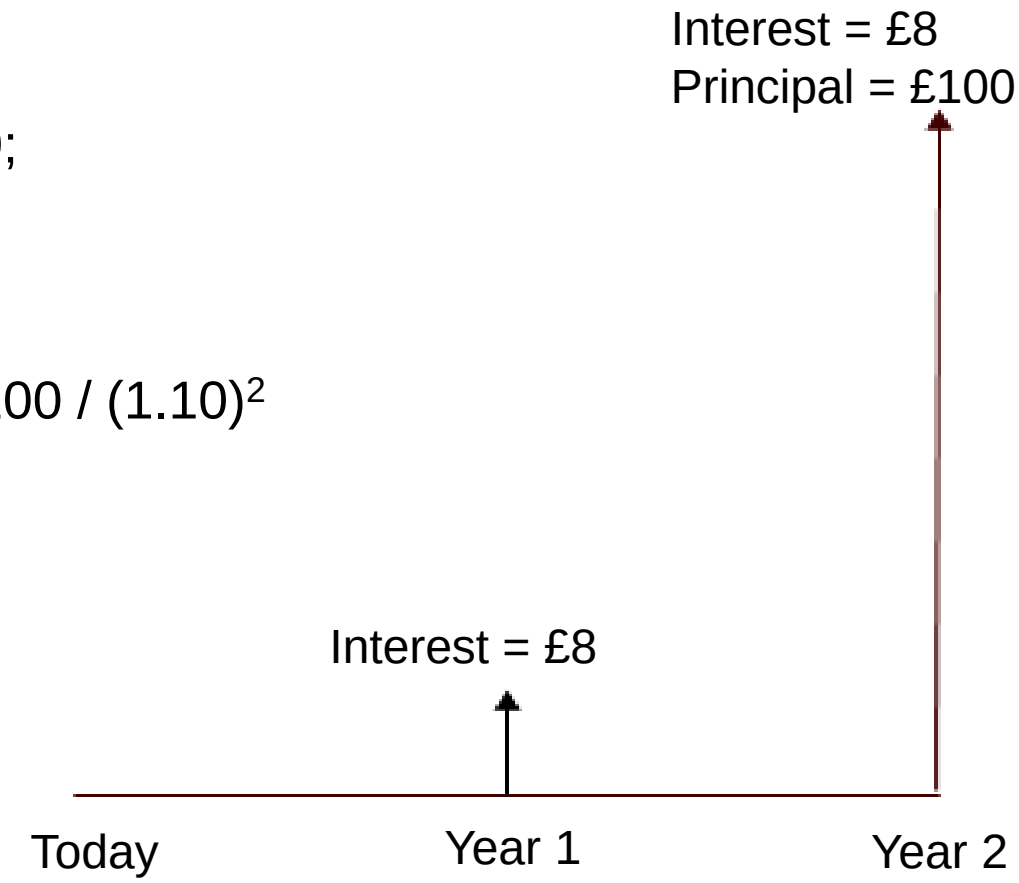
Present value

$= £8 / (1.10)^1 + £8 / (1.10)^2 + £100 / (1.10)^2$

$= £7.27 + £6.61 + £82.64$

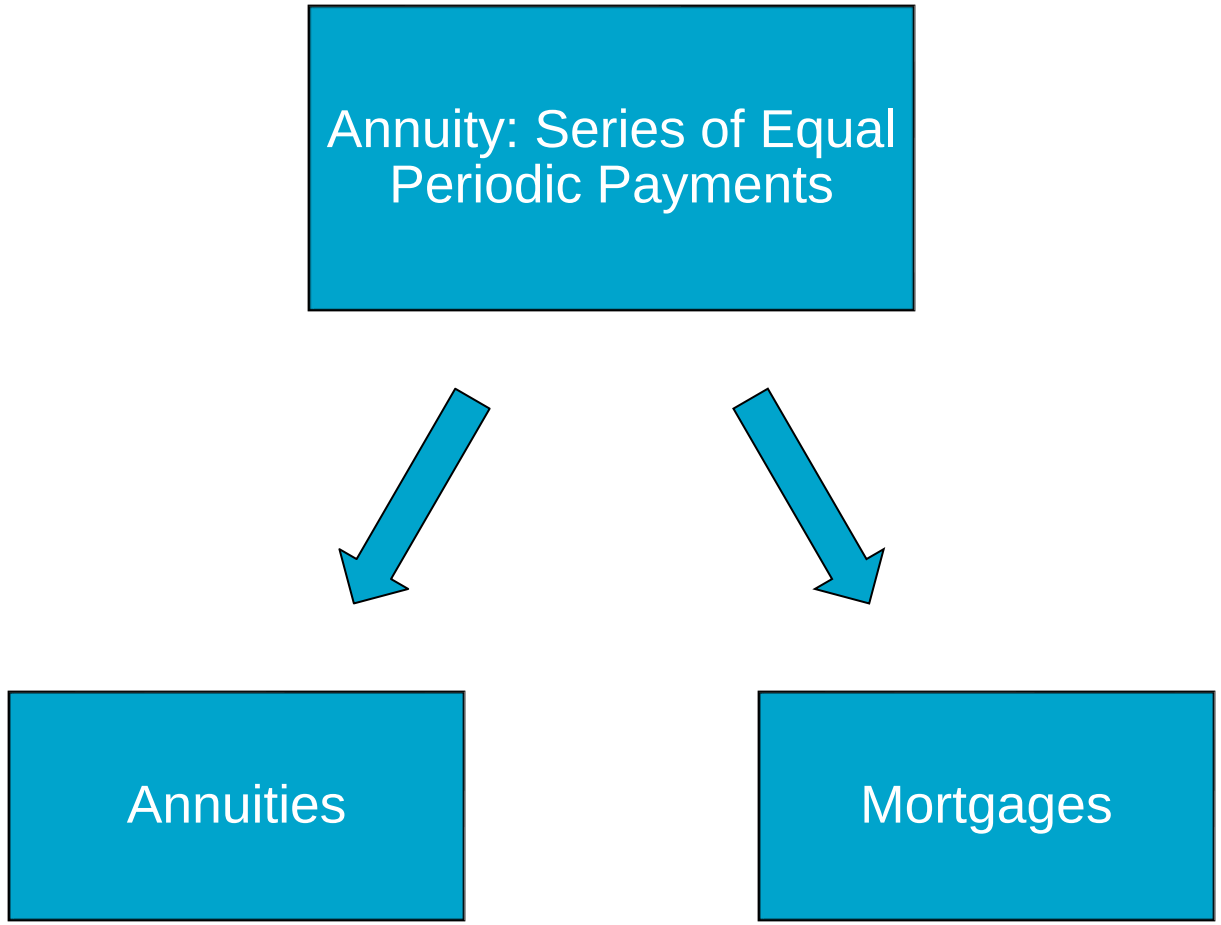
$= £96.52$

Note: Earns 8%, discounted at 10%, present value is less than principal



LOS f: Describe applications of time value of money.

ANNUITIES AND MORTGAGES



LOS f: Describe applications of time value of money.

ANNUITIES AND MORTGAGES

You borrow £60,000 to buy a small cottage in the country.
The interest rate on the mortgage is 4.60%. Your payment at the end of each year will be £13,706 for five years.

Year	Mortgage Outstanding at Beginning of Year	Total Mortgage Payment	Interest Paid	Principal Reduced by
1	£60,000	£13,706	£2,760 (= 60,000 × 0.046)	£10,946
2	£49,054	£13,706	£2,257 (= 49,054 × 0.046)	£11,449
3	£37,605	£13,706	£1,730 (= 37,605 × 0.046)	£11,976
4	£25,630	£13,706	£1,179 (= 25,630 × 0.046)	£12,527
5	£13,103	£13,706	£603 (= 13,103 × 0.046)	£13,103
6	£0			

Note: The LOS says DESCRIBE, NOT CALCULATE!

LOS f: Describe applications of time value of money.

PRACTICE Q:DIFFICULT

In a mortgage transaction, the amount of each fixed payment made by the borrower that represents interest:

A. decreases over time.
B. remains the same over time.
C. increases over time.

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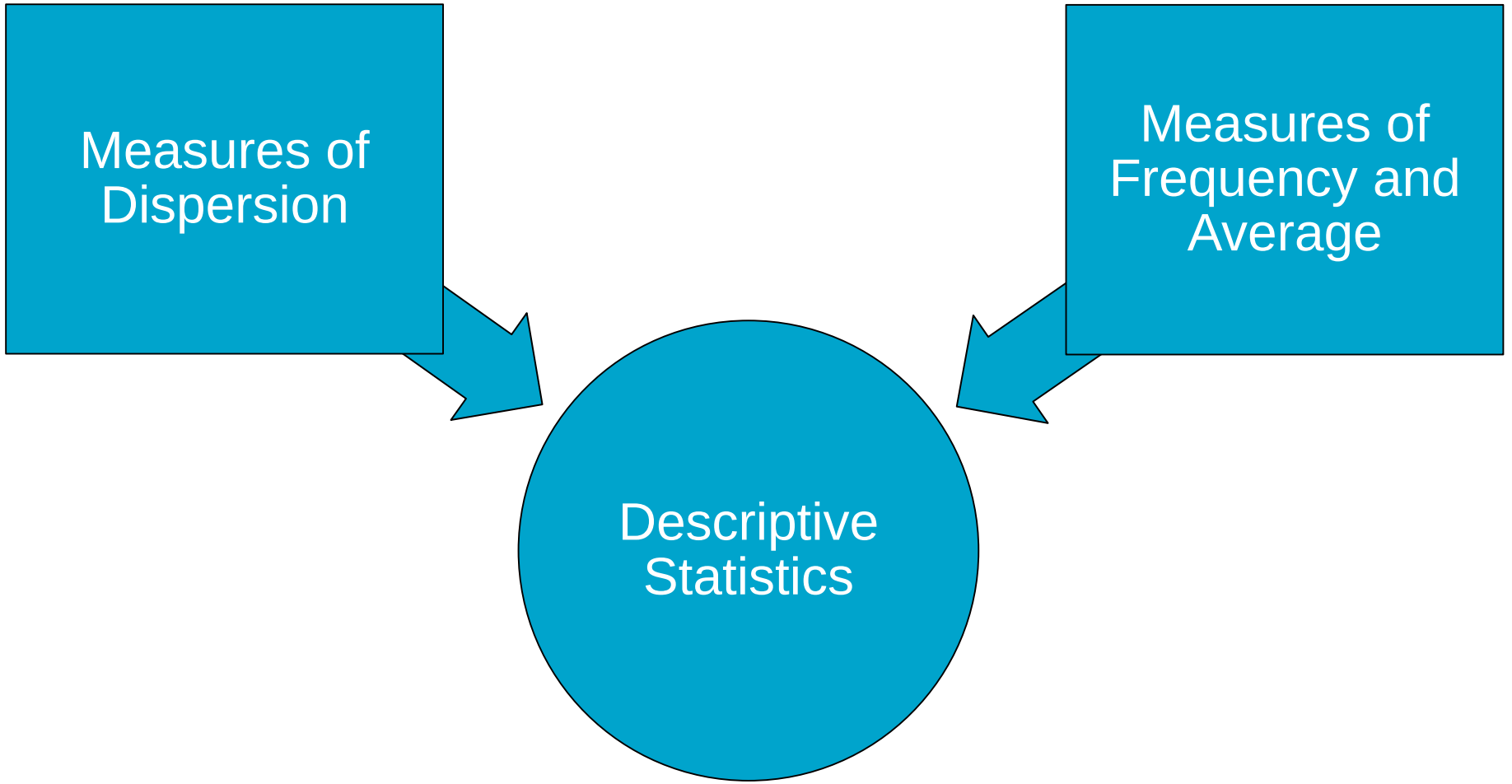
5	£13,103	£13,706	£603 (= 13,103 × 0.046)	£13,103
6	£0			

A is correct. Although the payment amount is fixed, the portion of each payment that is interest is based on the remaining principal at the beginning of each period.

As the principal declines, so does the amount of the fixed payment that constitutes interest.

DESCRIPTIVE STATISTICS



LOS g: Explain uses of mean, median, and mode, which are measures of frequency or central tendency.

ARITHMETIC MEAN, MEDIAN, AND MODE

Year	Annual Return Chronologically	Annual Return Ordered Low to High
1	1.3%	0.8%
2	2.4	1.3
3	0.8	2.4
4	3.7	3.7
5	8.0	3.7
6	3.7	4.2
7	7.2	5.2
8	26.4	7.2
9	4.2	8.0
10	5.2	26.4
Sum =	63.0%	

Note: If the 26.4% value is removed, the arithmetic mean drops to

$$4.1\% = \frac{(63.0\% - 26.4\%)}{9}$$

Arithmetic mean = “average” = 63.0% / 10 = 6.3%

LOS g: Explain uses of mean, median, and mode, which are measures of frequency or central tendency.

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5	8.0	3.7
6	3.7	4.2
7	7.2	5.2
8	26.4	7.2
9	4.2	8.0
10	5.2	26.4
Sum =	63.0%	

Mode: Most Frequent

Median: In the Middle

If you put data in ascending order of size from the smallest to the largest, the median is the middle value.
If there is an even number of items in a data set, then you average the two middle observations.

$$\text{Median} = \frac{(3.7 + 4.2)}{2} \approx 4.0\%$$

LOS g: Explain uses of mean, median, and mode, which are measures of frequency or central tendency.

MEASURES OF CENTRAL TENDENCY

The Median

The median is the value of the middle item of a set of items that has been sorted into ascending or descending order. In an odd-numbered sample of n items, the median occupies the (n + 1)/2 position. In an even-numbered sample, we define the median as the mean of the values of items occupying the two middle items.

Stock	Consensus Current EPS	Consensus Current P/E
Caterpillar. Inc. (NYSE: CAT)	6.34	13.15
Ford Motor Company (NYSE: F)	1.55	10.97
General Dynamics (NYSE: GD)	6.96	12.15
Green Mountain Coffee Roasters (NASDAQ: GMCR)	3.25	25.27
McDonald's Corporation (NYSE: MCD)	5.61	17.16
Qlik Technologies NASDAQ: QLIK)	0.17	204.82
Questcor Pharmaceuticals (NASDAQ: QCOR)	4.79	13.94

The P/Es listed in ascending order are:

10.97 12.15 13.15 13.94 17.16 25.27 204.82

The sample has an odd number of observations with n = 7, so the median occupies the (n+1)/2 = 8/2 = 4th position in the sorted list.
Therefore, the median P/E is 13.94.

PRACTICE Q: EXPERT

One of the numbers in a dataset happens to be the median of the sample. Which of the following conditions could give rise to this situation?

A. The median value is also the average.
B. There is an odd number of items in the dataset.
C. There is an even number of items in the dataset and the two middle-ranked numbers are different in value.

PRACTICE Q: EXPERT

One of the numbers in a dataset happens to be the median of the sample. Which of the following conditions could give rise to this situation?

A. The median value is also the average.

B. There is an odd number of items in the dataset.

C. There is an even number of items in the dataset and the two middle-ranked numbers are different in value.

B is correct. For the median to be one of the numbers in a dataset, there must be an odd number of items in the dataset, or if there is an even number of items in the dataset, the two middle values must be the same. In the latter case, summing the two middle values and dividing by two will return the same value.

MEASURES OF CENTRAL TENDENCY

The Mode

The mode is the most frequently occurring value in a distribution

A distribution can have more than one mode, or even no mode. When a distribution has **one most frequently occurring value**, the distribution is said to be **unimodal**.

If a distribution has **two most frequently occurring values**, then it has two modes and we say it is **bimodal**. If the distribution has three most frequently occurring values, then it is trimodal. **When all the values in a data set are different, the distribution has no mode because no value occurs more frequently than any other value**.

Stock return data and other data from continuous distributions may not have a modal outcome. When such data are grouped into intervals, however, we often find an interval (possibly more than one) with the highest frequency: **the modal interval** (or intervals).

GEOMETRIC MEAN

$$\text{Geometric mean} = [(1 + r_1) \times \ldots (1 + r_t)]^{1/t} - 1$$

Geometric mean

$= [(1 + 1.3\%) \times (1 + 2.4\%) \times (1 + 0.8\%)$

$\times (1 + 3.7\%) \times (1 + 8.0\%) \times (1 + 3.7\%)$

$\times (1 + 7.2\%) \times (1 + 26.4\%) \times (1 + 4.2\%)$

$\times (1 + 5.2\%)]^{(1/10)} - 1$

$= 6.1\%$

Year	Annual Return Chronologically
1	1.3%
2	2.4
3	0.8
4	3.7
5	8.0
6	3.7
7	7.2
8	26.4
9	4.2
10	5.2
Sum =	63.0%

Arithmetic mean = “average” = 63.0% / 10 = 6.3%

LOS g: Explain uses of mean, median, and mode, which are measures of frequency or central tendency.

ARITHMETIC OR MEAN RETURN

Geometric Mean Return

When looking at historical returns on investments we want to use geometric averages. This quick example will show why. In this next example we started with USD$10,000 and had the following returns for four years: +10%, -25%, +21%, +6%. The arithmetic average return is 3%. If we used that and calculated USD $10,000 x 1.03 x 1.03 x 1.03 x 1.03 we get a total of USD$11,255.08, but that is not what our actual balance would be. Let us do the math again:

- $10,000 x 10% = $11,000 at the end of year 1
- 11,000 x -25% = $8,250 at the end of year 2
- 8,250 x 21% = $9,982.50 at the end of year 3.
- $9,982.5 x 6% = **$10,581.45** at the end of year 4.

ARITHMETIC OR MEAN RETURN

Let us now calculate the geometric average using the following formula:

$$\sqrt[n]{(1 + r_1)x\ (1 +\ r_2)\ x\ \ldots..(1 + r_n)}$$

Or we could rewrite the formula as ((1+ r_1) x (1 + r_2) x …..(1 + r_n))^1/n – 1 x 100

Geometric average = (1+.10) x (1-.25) x (1+.21) x (1+.06)^1/4 – 1 x 100

= (1.1 x 0.75 x 1.21 x1.06)^1/4 – 1 x 100 = 1.42296%

Now use the geometric average: USD$10,000 x 1.0142296 x 1.0142296 x 1.0142296 x 1.0142296 = USD **$10,581.45**.

That is the correct balance in the account after four years. Using the geometric average adjusts for the fact that after a negative year there is less balance in the account on which to apply the following year's return. It is important to not use arithmetic averages when looking at average annual investment rates of returns.

RANGE AND PERCENTILES

Range = Highest value – Lowest value
= 0.264 – 0.008 = 0.2560 = 25.6%
Note: if the highest return, year 8, of 26.4% is removed, range = 8.0% - 0.8% = 7.2%

Year	Annual Return Ordered Low to High
1	0.8%
2	1.3
3	2.4
4	3.7
5	3.7
6	4.2
7	5.2
8	7.2
9	8.0
10	26.4
Sum =	63.0%

Percentiles: If there are a large number of observations ranked in order of size, the range can be divided into 100 equal-sized intervals.

- The dividing points are termed percentiles.
- **The 50th percentile is the median** and divides the observations so that 50% are higher and 50% are lower than the median.
- The 20th percentile is the value **below which 20% of observations in the series fall.**

LOS h: Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion.

PRACTICE Q: DIFFICULT

On a recent examination involving several hundred candidates, Dave's score was in the 75th percentile, Mary's was in the 15th percentile, and Pablo's was in the 20th percentile. Which of the following statements is correct?

A. Mary had the highest score.

B. Pablo earned the median score.

C. Dave's score was above the median.

PRACTICE Q: DIFFICULT

On a recent examination involving several hundred candidates, Dave's score was in the 75th percentile, Mary's was in the 15th percentile, and Pablo's was in the 20th percentile. Which of the following statements is correct?

A. Mary had the highest score.

B. Pablo earned the median score.

C. Dave's score was above the median.

C is correct. The 50th percentile is the value that divides the observations, with 50% higher and 50% lower, and is thus the median.

The 75th percentile is the value below which 75% percent of the observations in the series fall. Dave's score is thus the highest (75% of the others are lower) and is above the median (50%).

A is incorrect. The lowest score would be in the 15th percentile.

B is incorrect. The median score is the 50% percentile which Pablo did not achieve.

STEPS TO CALCULATE VARIANCE

Step 1: Calculate or use the return given for each period and sum up the returns

Step 2: Calculate the average return (Sum of returns divided by number of returns)

Step 3: Subtract the average return from each period return

Step 4: Square the result

Step 5: Add the sum of the squares

Step 6: Divide by the number of periods minus one (sample); **this gives us the variance**

Step 7: Take the square root of the variance and you have the standard deviation

Population	Sample
$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \mu)^2}{n}$	$s^2 = \frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{n-1}$
$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$

$$\text{Standard deviation} = \sqrt{\frac{[X_1 - E(X)]^2 + [X_2 - E(X)]^2 + \ldots + [X_n - E(X)]^2}{n}}$$

where

X_i = observation i (one of n possible outcomes for X)

n = number of observations of X

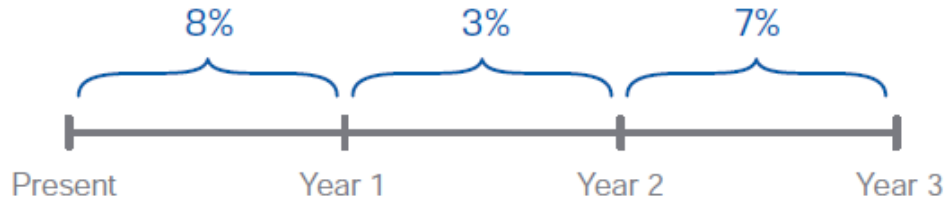
$E(X)$ = the mean (average) value of X or the expected value of X

$[X_i - E(X)]$ = difference between value of observation X_i and the mean value of X

LOS h: Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion.

STEPS TO CALCULATE VARIANCE

To illustrate the calculation of the standard deviation for an example of a three-year investment that returns 8% or 0.08 the first year, 3% or 0.03 the second year, and 7% or 0.07 the third year. The arithmetic mean return is 6% or 0.06. The standard deviation is approximately 2.16%.



$$\text{Standard deviation} = \sqrt{\frac{(0.08 - 0.06)^2 + (0.03 - 0.06)^2 + (0.07 - 0.06)^2}{3}}$$

$$= \sqrt{\frac{(0.02)^2 + (-0.03)^2 + (0.01)^2}{3}}$$

$$= \sqrt{\frac{(0.0004) + (0.0009) + (0.0001)}{3}}$$

$$= \sqrt{\frac{(0.0014)}{3}} = 0.0216 = 2.16\%$$

$$\text{Standard deviation} = \sqrt{\frac{[X_1 - E(X)]^2 + [X_2 - E(X)]^2 + \ldots + [X_n - E(X)]^2}{n}}$$

where

X_i = observation i (one of n possible outcomes for X)

n = number of observations of X

$E(X)$ = the mean (average) value of X or the expected value of X

$[X_i - E(X)]$ = difference between value of observation X_i and the mean value of X

LOS h: Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion.

STANDARD DEVIATION

Population	Sample
$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \mu)^2}{n}$	$s^2 = \frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{n-1}$
$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$

$$\text{Standard deviation} = \sqrt{\frac{[X_2 - E(X)]^2 + [X_2 - E(X)]^2 + \ldots + [X_n - E(X)]^2}{n}}$$

Standard deviation
= Square root of {[(0.013 – 0.063)^2 + (0.024 – 0.063)^2
+ (0.008 – 0.063)^2 + (0.037 – 0.063)^2 + (0.080 – 0.063)^2
+ (0.037 – 0.063)^2 + (0.072 – 0.063)^2 + (0.264 – 0.063)^2
+ (0.042 – 0.063)^2 + (0.052 – 0.063)^2]/10}

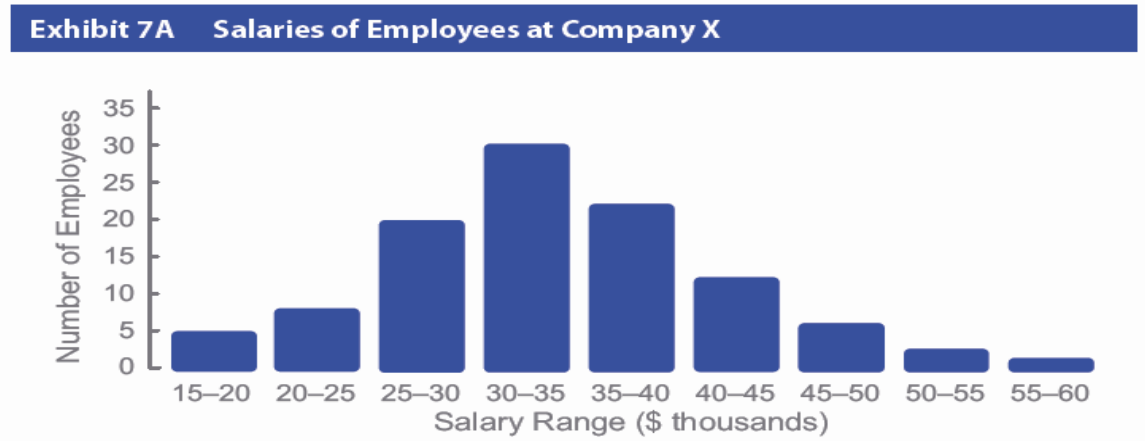
= Square root of [(0.0025 + 0.0015 + 0.0030 + 0.0007
+ 0.0003 + 0.0007 + 0.0001 + 0.0404 + 0.0004 + 0.0001)
/10] = 7.1%

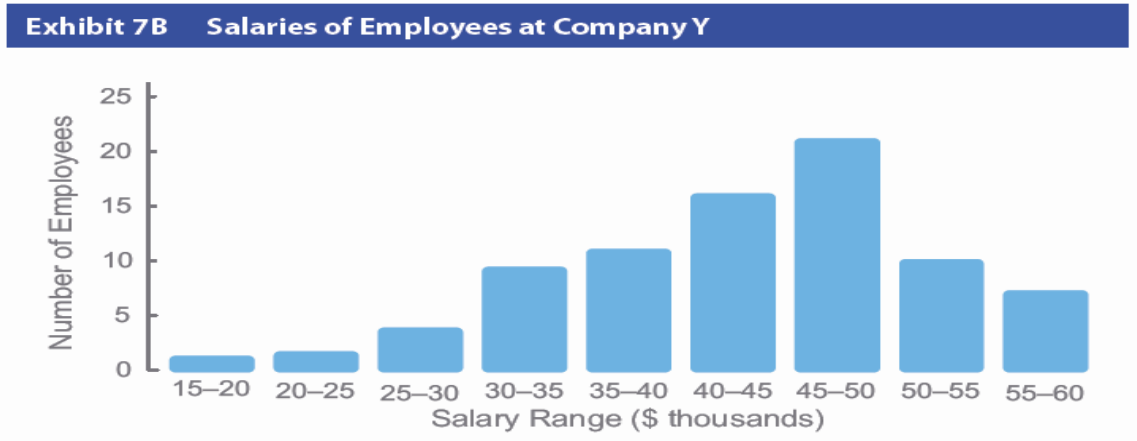
Note: Standard deviation is the square root of the variance.

Year	Annual Return Chronologically
1	1.3%
2	2.4
3	0.8
4	3.7
5	8.0
6	3.7
7	7.2
8	26.4
9	4.2
10	5.2
Sum =	63.0%

LOS h: Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion.

HISTOGRAMS





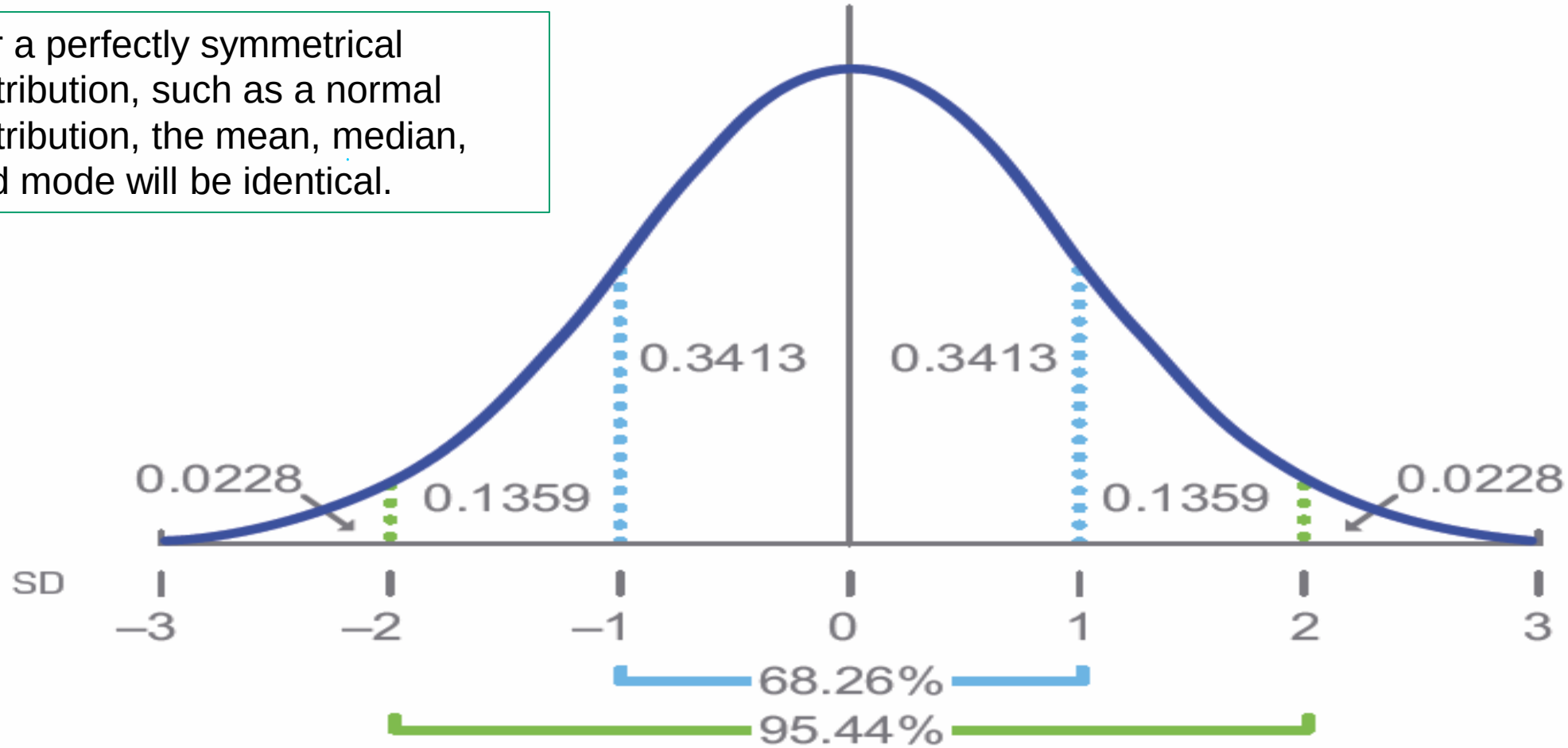
The distribution for Company X is positively skewed (i.e., the majority of the observations are on the left and the skew or tail is on the right), whereas the distribution for Company Y is negatively (left) skewed.

- If the distribution is skewed, the three measures of central tendency—mean, median, and mode—will differ.
- For a perfectly symmetrical distribution, such as a normal distribution, the mean, median, and mode will be identical.

LOS i: Describe and interpret the characteristics of a normal distribution;

NORMAL DISTRIBUTION

For a perfectly symmetrical distribution, such as a normal distribution, the mean, median, and mode will be identical.



LOS i: Describe and interpret the characteristics of a normal distribution;

PRACTICE Q: DIFFICULT

The proportion of the observations that fall within one standard deviation above and below the mean of a normal distribution is approximately:

A. 50%.

B. 68%

C. 95%.

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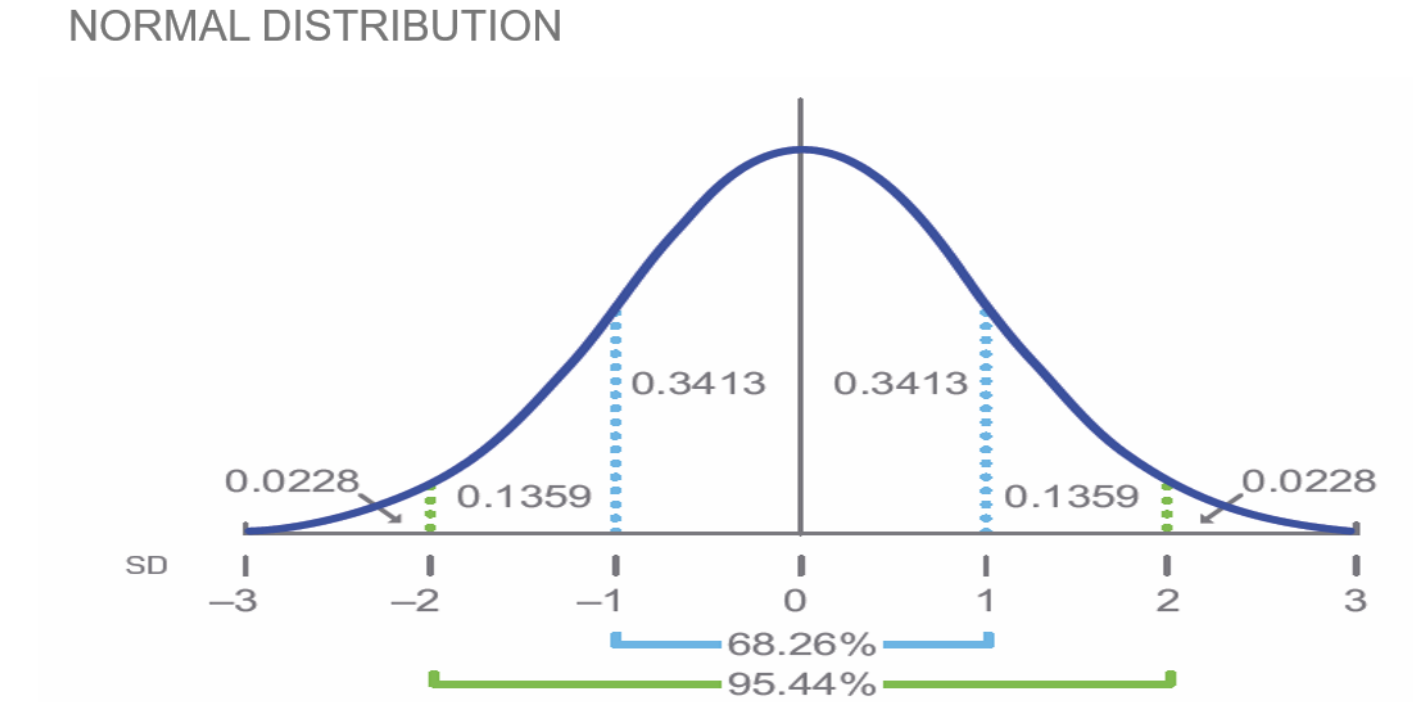
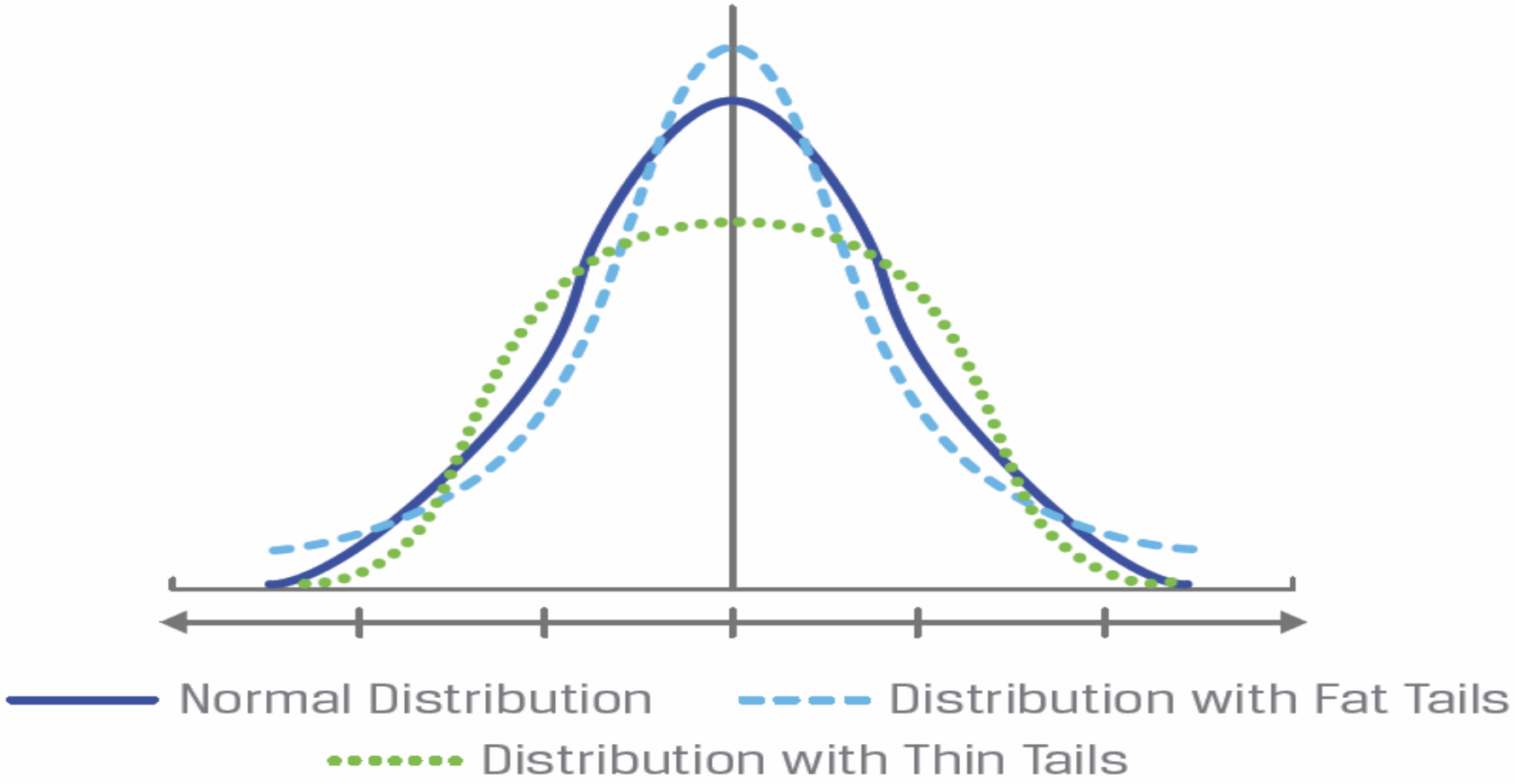


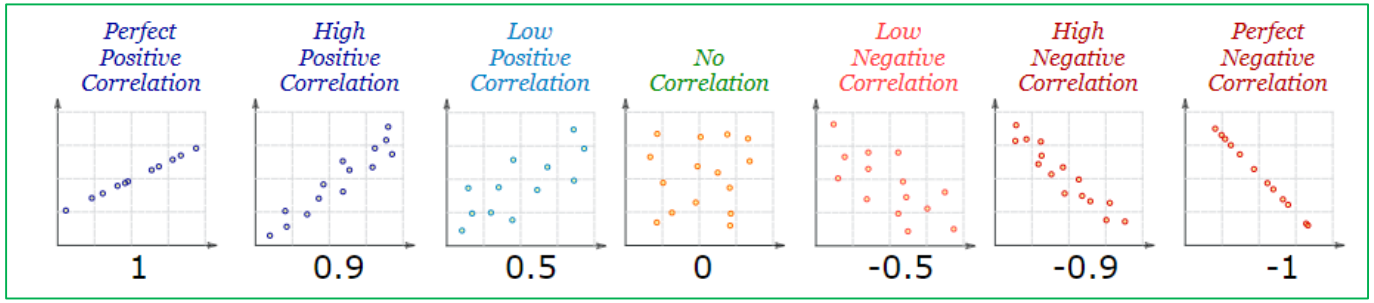
B is correct. 68% of the observations fall within one standard deviation of the mean of a normal distribution.

FAT AND THIN TAILS



LOS i: Describe and interpret the characteristics of a normal distribution;

CORRELATION



https://www.mathsisfun.com/data/correlation.html

Direction of the Relationship

If negative, tend to move in opposite directions; if positive, tend to move in same direction.

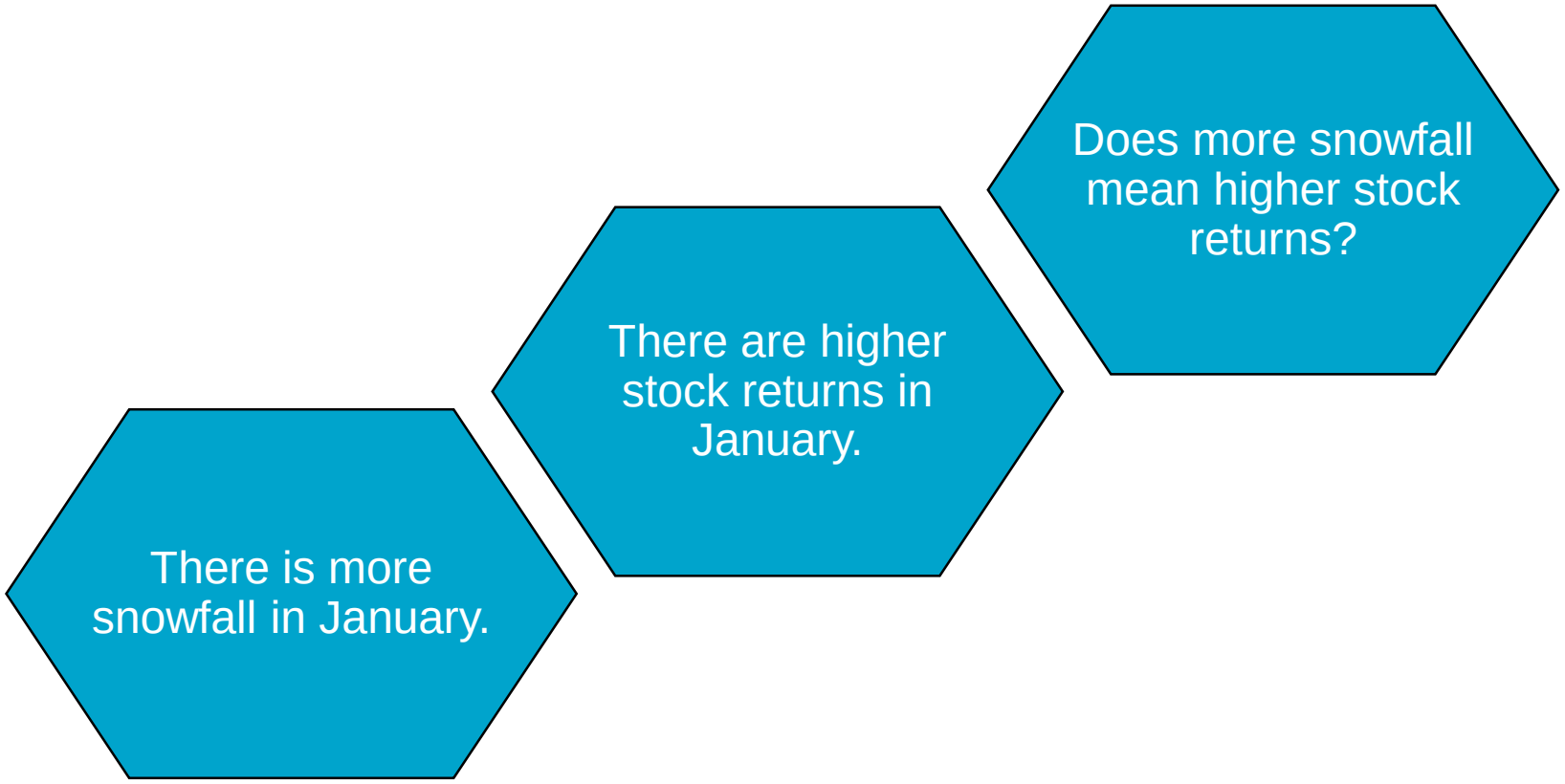
Strength of the Relationship

If close to zero, tend to be independent; if close to –1 or +1, strong relationship.

$$-1 \leq \text{Correlation} \leq +1$$

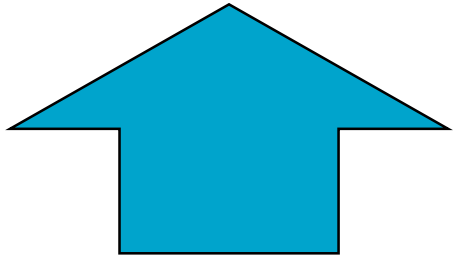
LOS j: Describe and interpret correlation.

CORRELATION VS. CAUSATION



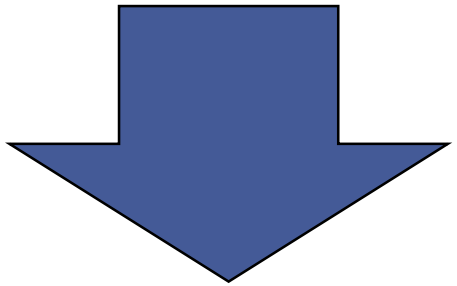
LOS j: Describe and interpret correlation.

DIVERSIFICATION



Adding Securities to a Portfolio

Reduces Risk



As long as the returns on the securities do not have a correlation of +1 (that is, they are less than perfectly correlated), then the risk of the portfolio will be less than the weighted average of the risks of the securities in the portfolio because it is not likely that all the securities will perform poorly at the same time.

LOS j: Describe and interpret correlation.

STEPS TOWARD AN ACTUAL PORTFOLIO

Correlations and volatility

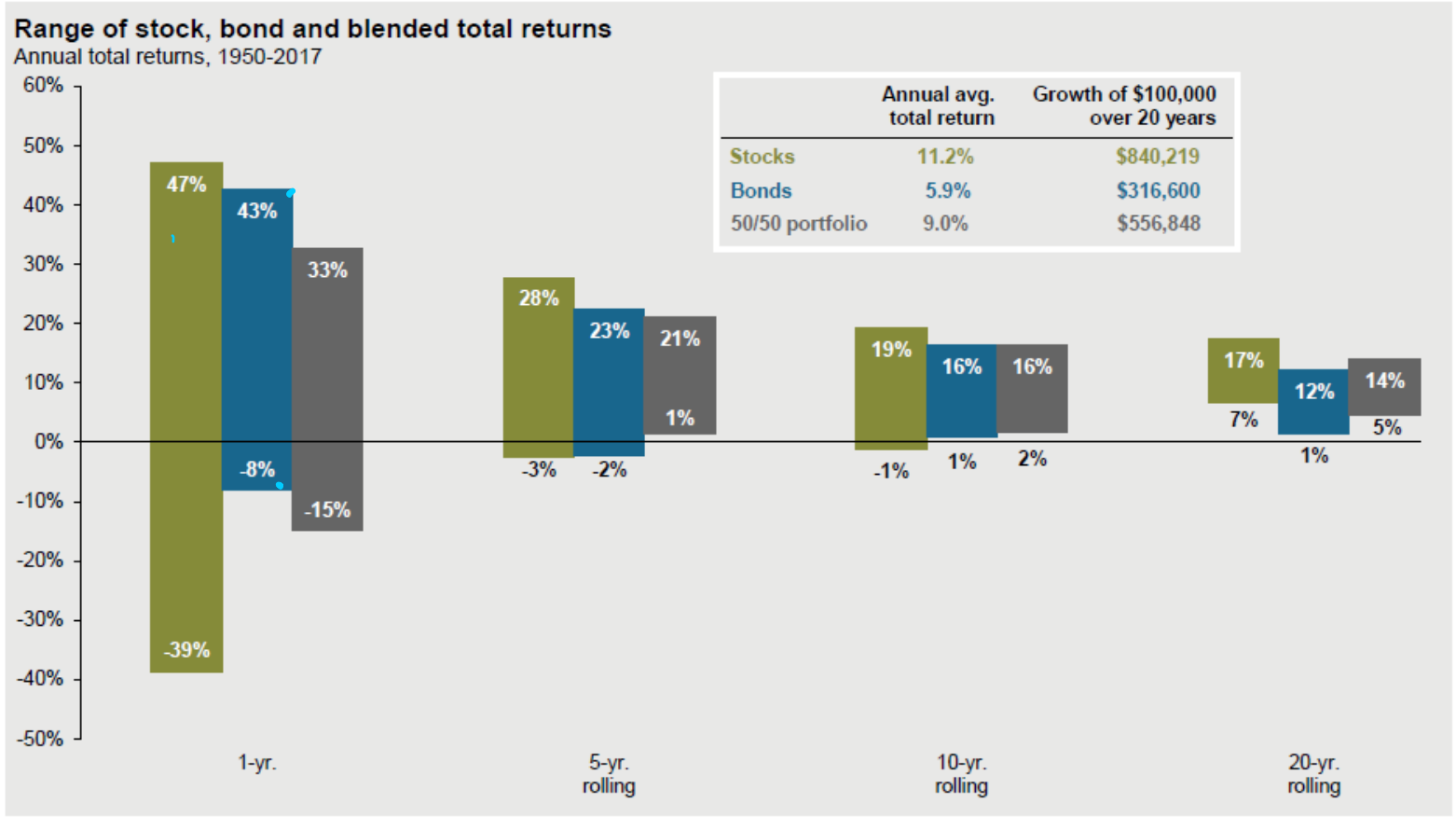
	U.S. Large Cap	EAFE	EME	Bonds	Corp. HY	Munis	Currcy.	EMD	Cmdty.	REITs	Hedge funds	Private equity	Ann. Volatility
U.S. Large Cap	1.00	0.89	0.79	-0.31	0.72	-0.18	-0.51	0.58	0.66	0.83	0.87	0.85	15%
EAFE		1.00	0.90	-0.17	0.77	-0.06	-0.67	0.69	0.64	0.75	0.85	0.79	18%
EME			1.00	-0.09	0.88	0.01	-0.70	0.84	0.70	0.66	0.85	0.73	22%
Bonds				1.00	-0.04	0.83	-0.12	0.27	-0.22	0.04	-0.29	-0.39	3%
Corp. HY					1.00	0.08	-0.53	0.87	0.71	0.72	0.83	0.68	12%
Munis						1.00	-0.14	0.43	-0.19	0.10	-0.12	-0.26	4%
Currencies							1.00	-0.61	-0.56	-0.44	-0.44	-0.54	7%
EMD								1.00	0.59	0.63	0.69	0.53	8%
Commodities									1.00	0.56	0.72	0.76	17%
REITs										1.00	0.71	0.74	25%
Hedge funds											1.00	0.84	6%
Private equity												1.00	10%

Source: Barclays Inc., Bloomberg, Cambridge Associates, Credit Suisse/Tremont, FactSet, Federal Reserve, MSCI, NCREIF, Standard & Poor's, J.P. Morgan Asset Management.
Indices used – Large Cap: S&P 500 Index; Currencies: Federal Reserve Trade Weighted Dollar; EAFE: MSCI EAFE; EME: MSCI Emerging Markets; Bonds: Bloomberg Barclays Aggregate; Corp HY: Bloomberg Barclays Corporate High Yield; EMD: Bloomberg Barclays Emerging Market; Cmdty.: Bloomberg Commodity Index; Real Estate: NAREIT ODCE Index; Hedge Funds: CS/Tremont Hedge Fund Index; Private equity: Cambridge Associates Global Buyout & Growth Index. Private equity data are reported on a one-quarter lag. All correlation coefficients and annualized volatility are calculated based on quarterly total return data for period 9/30/08 to 9/30/18, except for Private equity, which is based on the period from 6/30/08 to 6/30/18. This chart is for illustrative purposes only.
Guide to the Markets – U.S. Data are as of September 30, 2018.

J.P.Morgan
Asset Management

Source: (Morgan, JP 2018)

STEPS TOWARD AN ACTUAL PORTFOLIO



Source: Barclays, Bloomberg, FactSet, Federal Reserve, Robert Shiller, Strategas/Ibbotson, J.P. Morgan Asset Management.
Returns shown are based on calendar year returns from 1950 to 2017. Stocks represent the S&P 500 Shiller Composite and Bonds represent Strategas/Ibbotson for periods from 1950 to 2010 and Bloomberg Barclays Aggregate thereafter. Growth of $100,000 is based on annual average total returns from 1950 to 2017.
Guide to the Markets – U.S. Data are as of September 30, 2018.

J.P.Morgan
Asset Management

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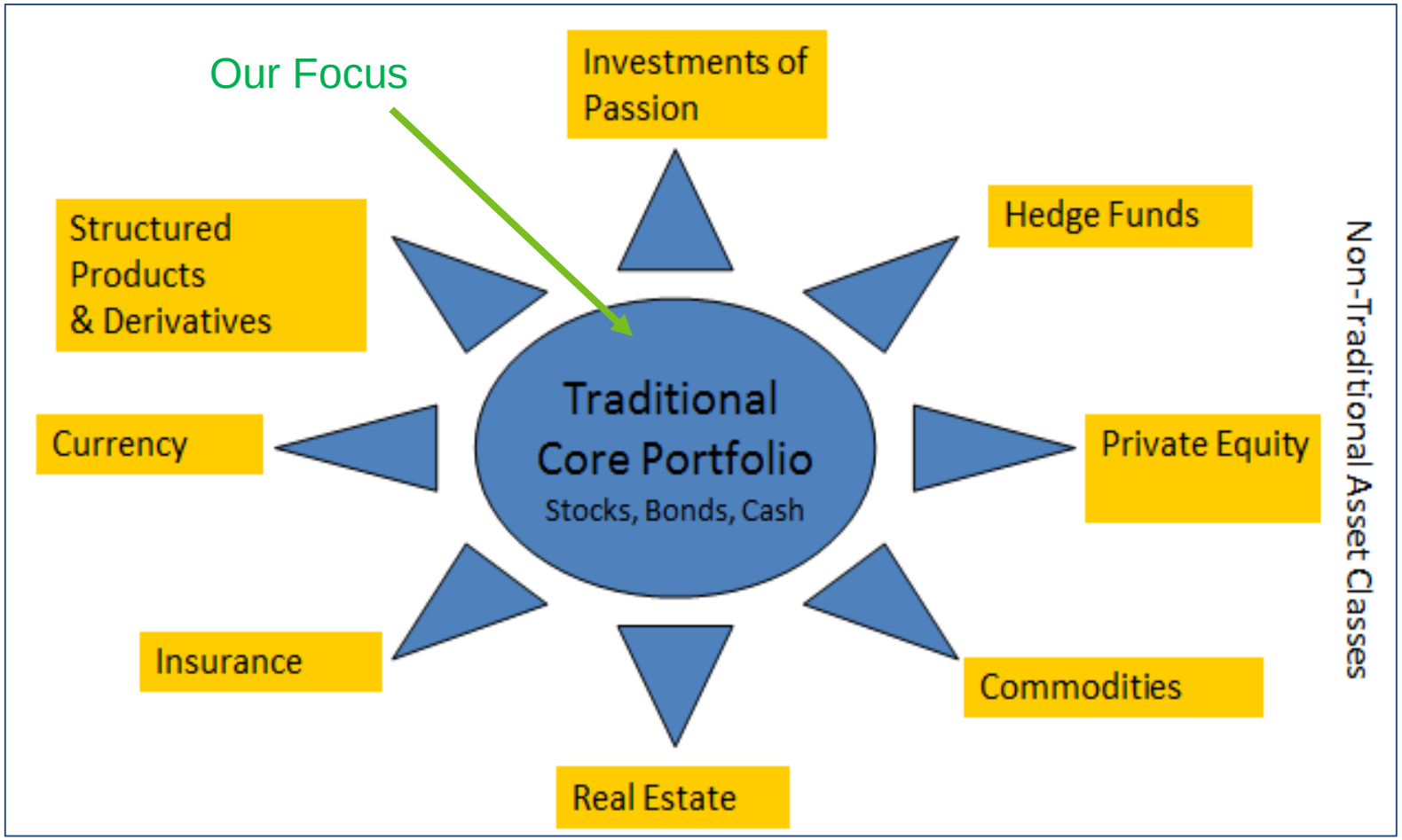
CFA Institute®
Investment Foundations

EXTRA SLIDES

STEPS TOWARD AN ACTUAL CORE PORTFOLIO

Core-Satellite Investing



STRATEGIC ASSET ALLOCATION – JACK BOGLE ON SIMPLICITY

Stick to simplicity.

- "Don't complicate the process. Basic investing is simple—a sensible asset allocation to **stocks, bonds, and cash**."

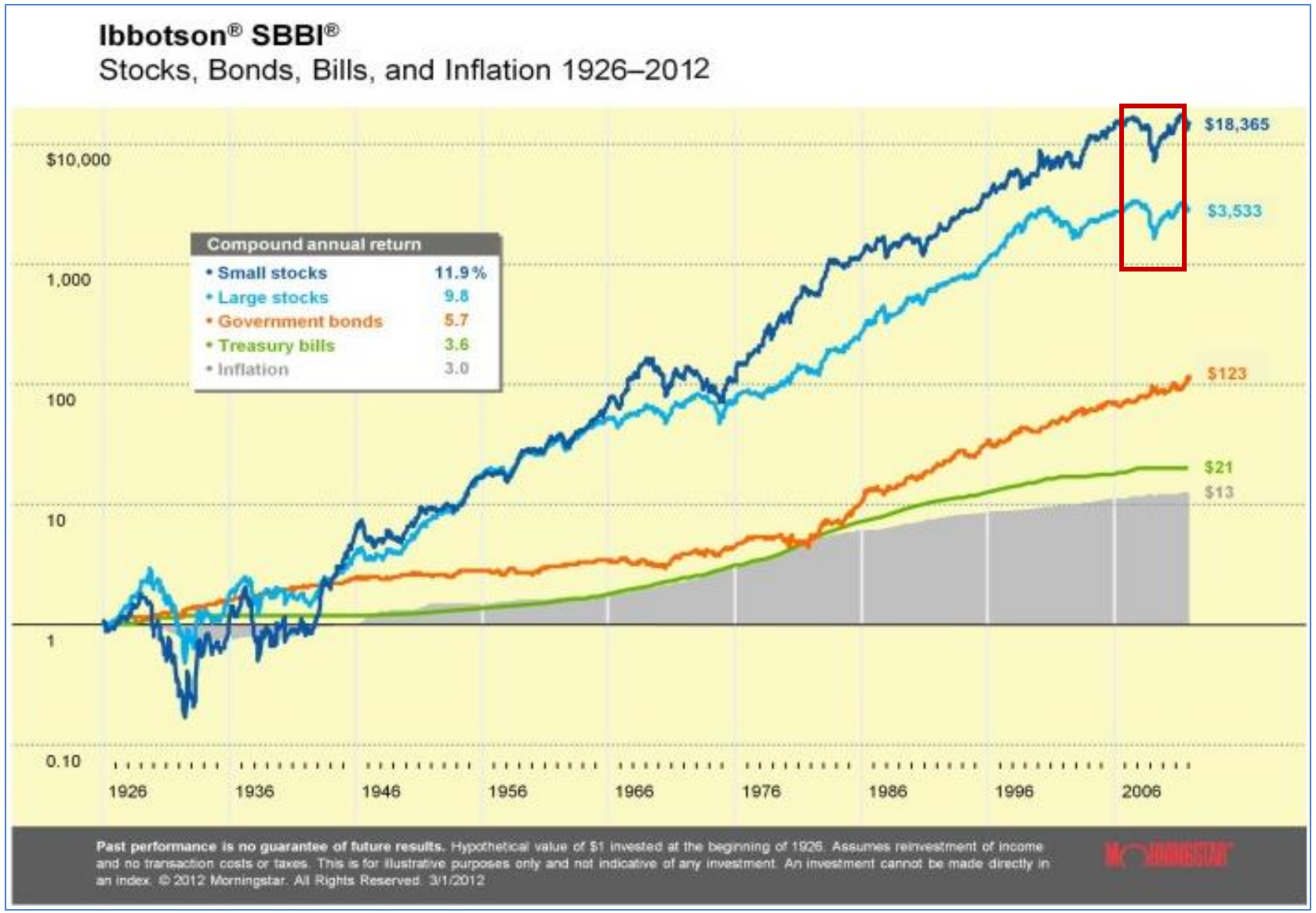
- Perhaps the most critical decision you face is getting **the proper allocation of assets** in your investment portfolio.

- **Stocks are designed to provide growth of capital and growth of income, while bonds are for conservation of capital and current income.**

- Once you get your balance right, then just hold tight, no matter how high a greedy stock market flies, nor how low a frightened market plunges.

- Change the allocation only as your investment profile changes. **The paradox is that in these times of increasing complexity, simplicity underlies the best investment strategy**." (Bogle 2012)

GROWTH OF $1: 1926 – 2012*: USA DATA



Stocks

Bonds

T Bills

Cash

RISKS IN INVESTING: DEFINITION OF RISK

- Greater variability in market prices and cash flows can be thought of as commensurate with increased risk because an investor owning a risky asset with a highly variable price pattern faces having to sell it for a more unpredictable price than a less risky asset.

- The assumption that variability in asset returns represents risk and that premiums over what could be earned on a risk-less investment represent the price of risk is the foundation for modern finance theory.

- **It should be stressed that when analyzing investments, returns provide us with only one half the information we need. Information on the risk characteristics of investments are equally important**.

- We measure risk by calculating variance and standard deviation

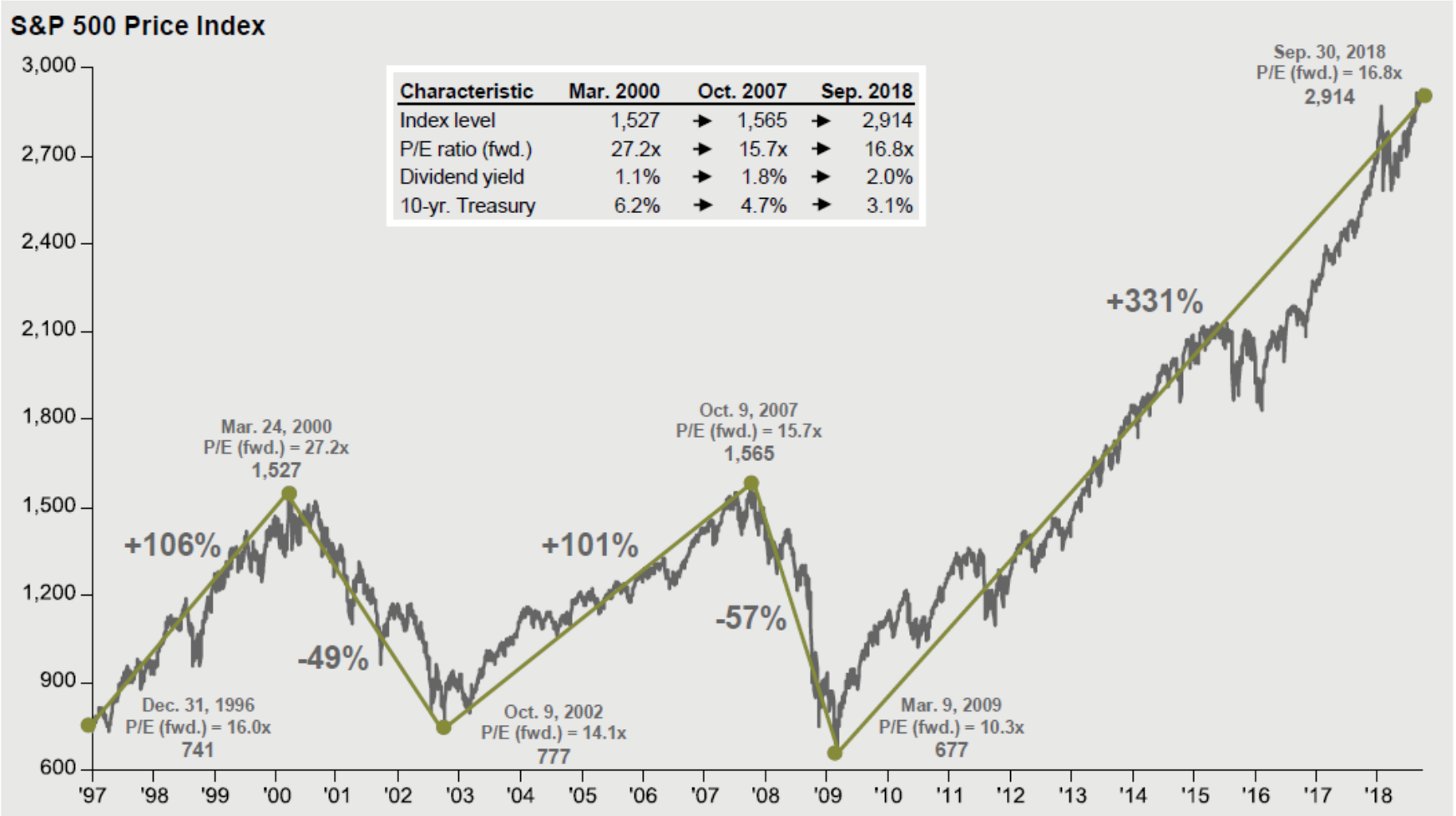
HISTORICAL MEAN RETURN AND EXPECTED RETURN

1926–2012 Returns RISK

	Asset Class	Annual Returns	Standard Deviation
STOCKS	Small-cap	11.9%	33.0%
	Large-cap	**9.8%**	20.9%
BONDS	LT Corporate Bonds	5.7%	9.4%
	LT Treasury Bonds	5.5%	9.0%
CASH	Treasury Bills	3.6%	3.1%
	Inflation	3.0%	4.2%

Source: 2012 Ibbotson SBBI Classic Yearbook

S&P 500

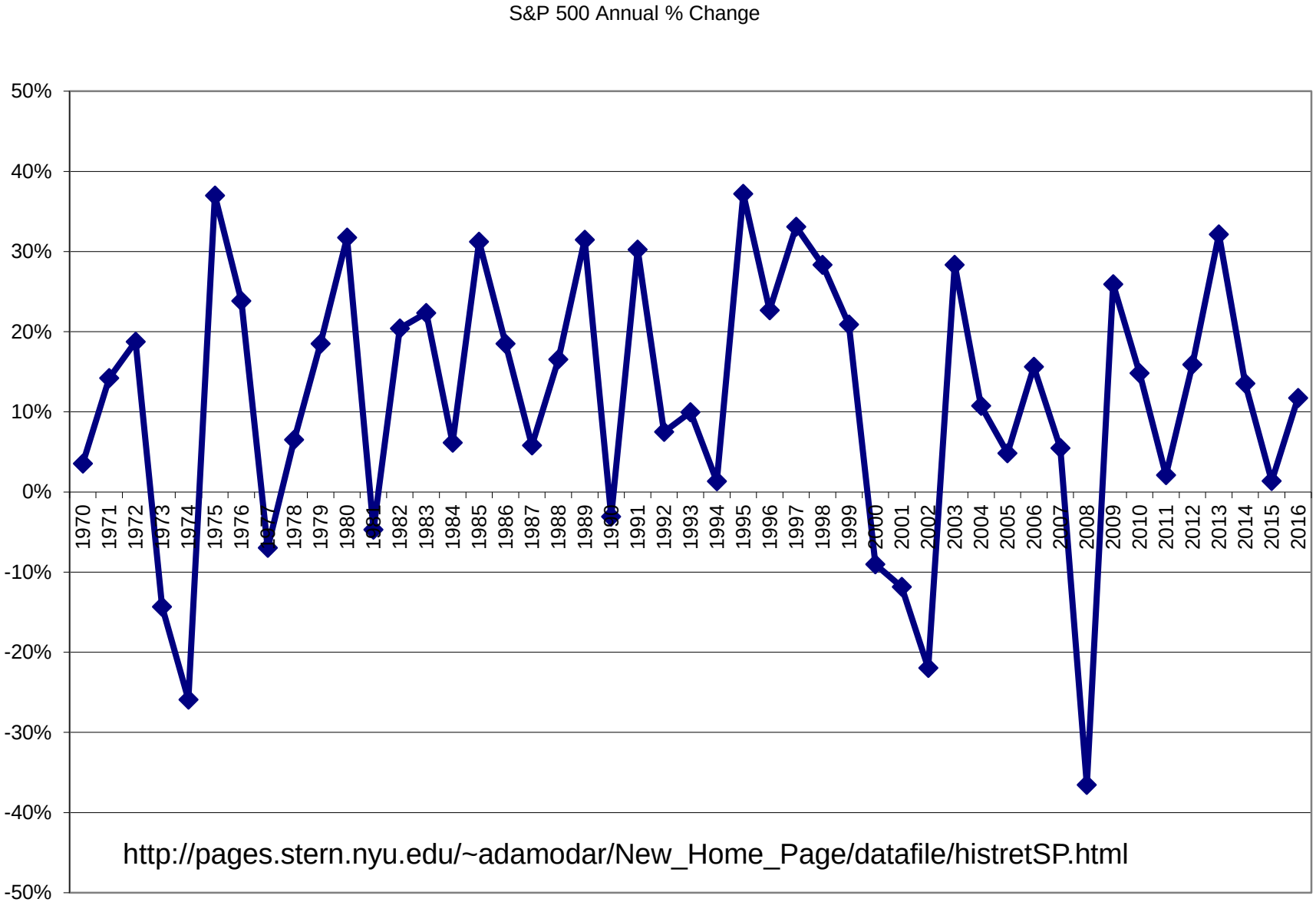


Source: Compustat, FactSet, Federal Reserve, Standard & Poor's, J.P. Morgan Asset Management.
Dividend yield is calculated as consensus estimates of dividends for the next 12 months, divided by most recent price, as provided by Compustat. Forward price to earnings ratio is a bottom-up calculation based on the most recent S&P 500 Index price, divided by consensus estimates for earnings in the next 12 months (NTM), and is provided by FactSet Market Aggregates. Returns are cumulative and based on S&P 500 Index price movement only, and do not include the reinvestment of dividends. Past performance is not indicative of future returns.
Guide to the Markets – U.S. Data are as of September 30, 2018.

2,506.85+21.11 (+0.85%)At close: December 31 5:05PM EST

J.P.Morgan
Asset Management

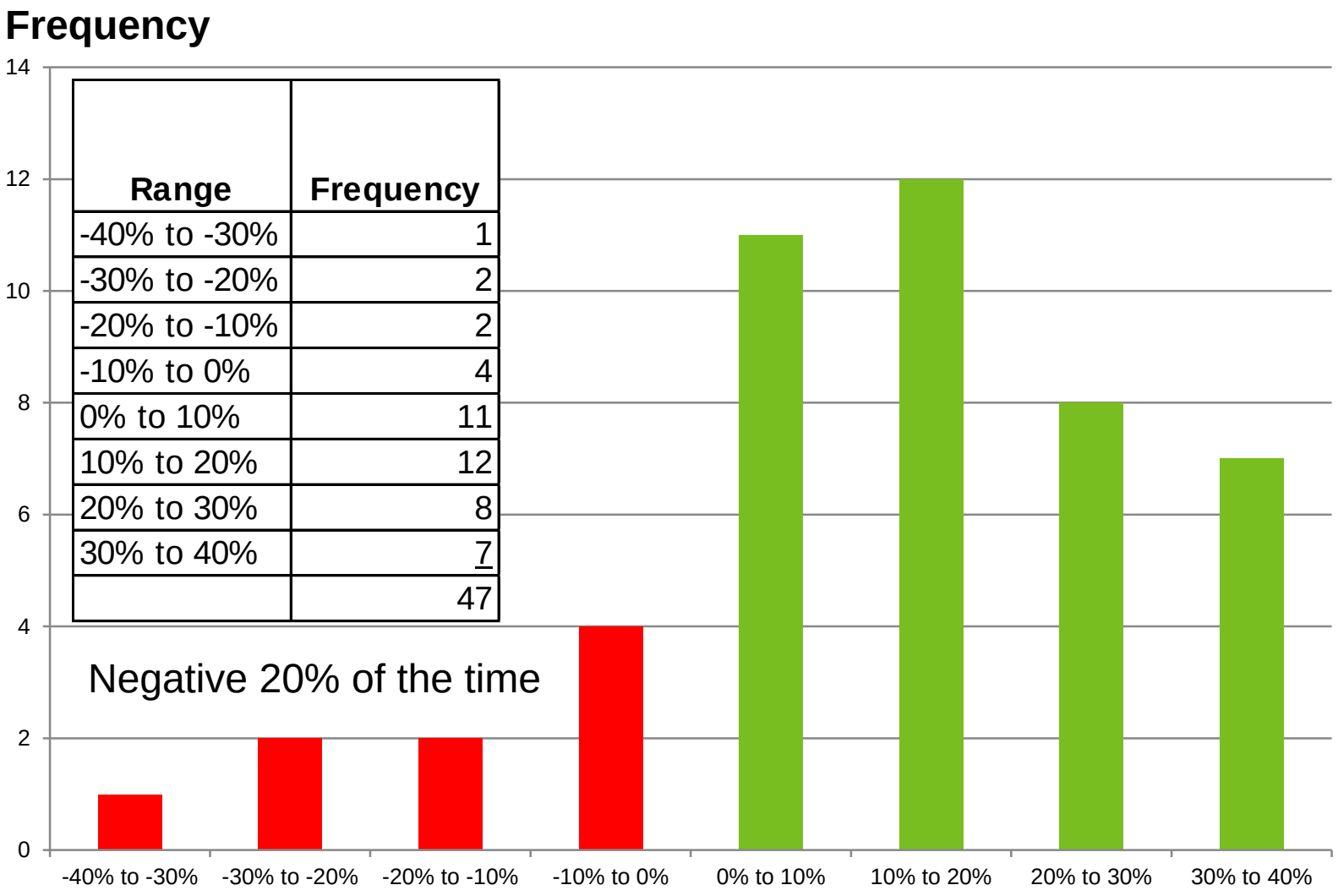
S&P 500 ANNUAL RETURNS 1970–2016 (INCLUDES DIVIDENDS)



2017: 21.64%
2018: Approx -7%

1970–2016 FREQUENCY OF S&P 500 ANNUAL RETURNS

Range	Frequency
-40% to -30%	1
-30% to -20%	2
-20% to -10%	2
-10% to 0%	4
0% to 10%	11
10% to 20%	12
20% to 30%	8
30% to 40%	7
	47



Stock return data and other data from continuous distributions may not have a modal outcome. When such data are grouped into intervals, however, we often find an interval (possibly more than one) with the highest frequency: the modal interval (or intervals). Ibbotson data: this was the 10% – 20% return.

BENJAMIN GRAHAM ON RISK

Benjamin Graham stated that**: "every investor who owns common stocks must expect to see them fluctuate in value over the years.**

If you overestimate how well you really understand an investment, or overstate your ability to ride out a temporary plunge in prices, it doesn't matter what you own or how the market does.

Ultimately, financial risk resides not in what kinds of investments that you have, but in what kind of investor you are." (Graham 2005)

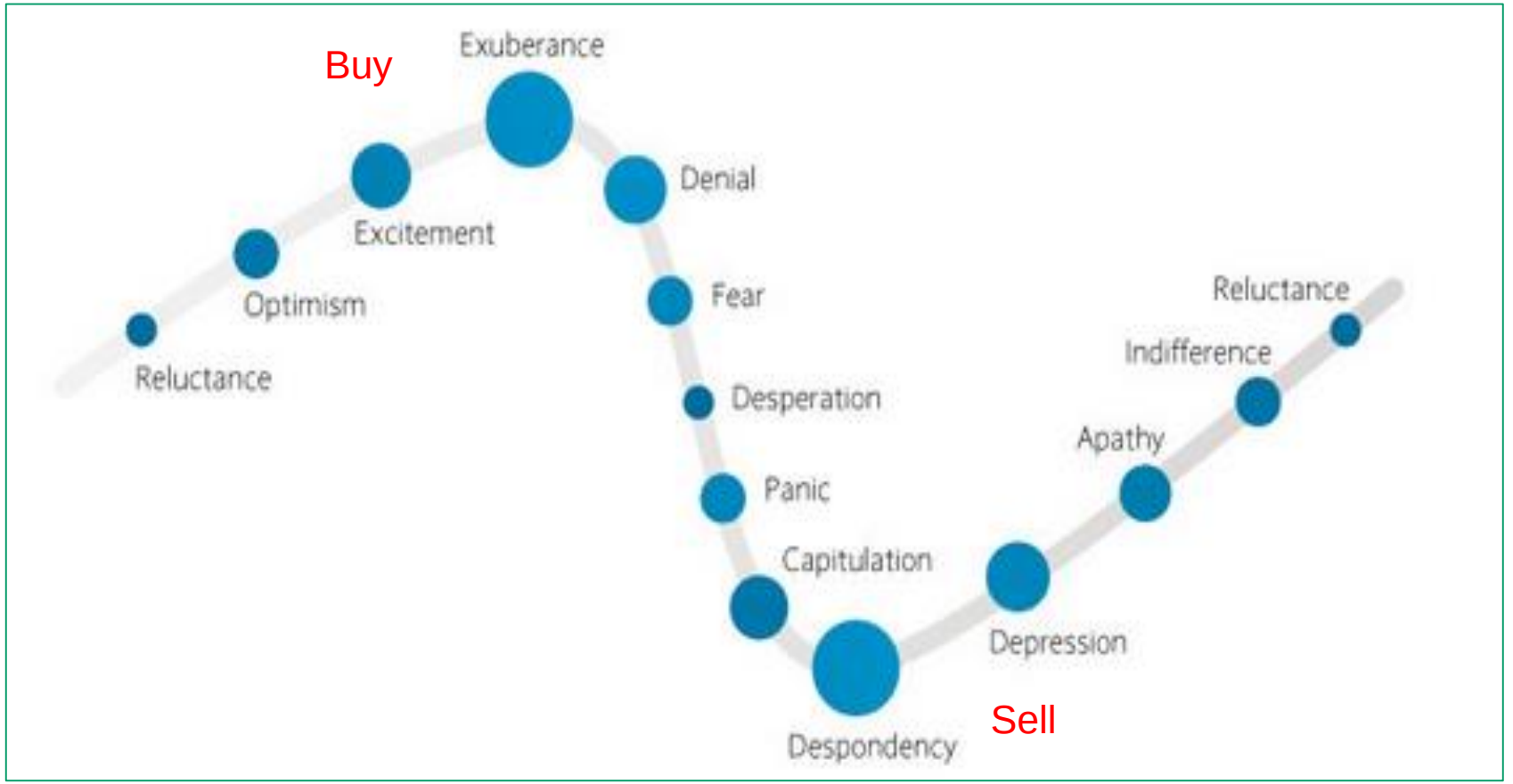
The market is a pendulum that swings between unsustainable optimism (which makes the stocks too expensive) and unjustified pessimism (which makes them too cheap). The intelligent investor is a realist who sells to optimists and buys from pessimists.

Warren Buffett echoed this advice when he said "be fearful when others are greedy and greedy when others are fearful," (Buffet 2008) and "The sillier the market's behavior, the greater the opportunity for the business-like investor."

Success comes from harnessing our emotions and understanding the things we can control and cannot control.

STEPS TOWARD AN ACTUAL PORTFOLIO

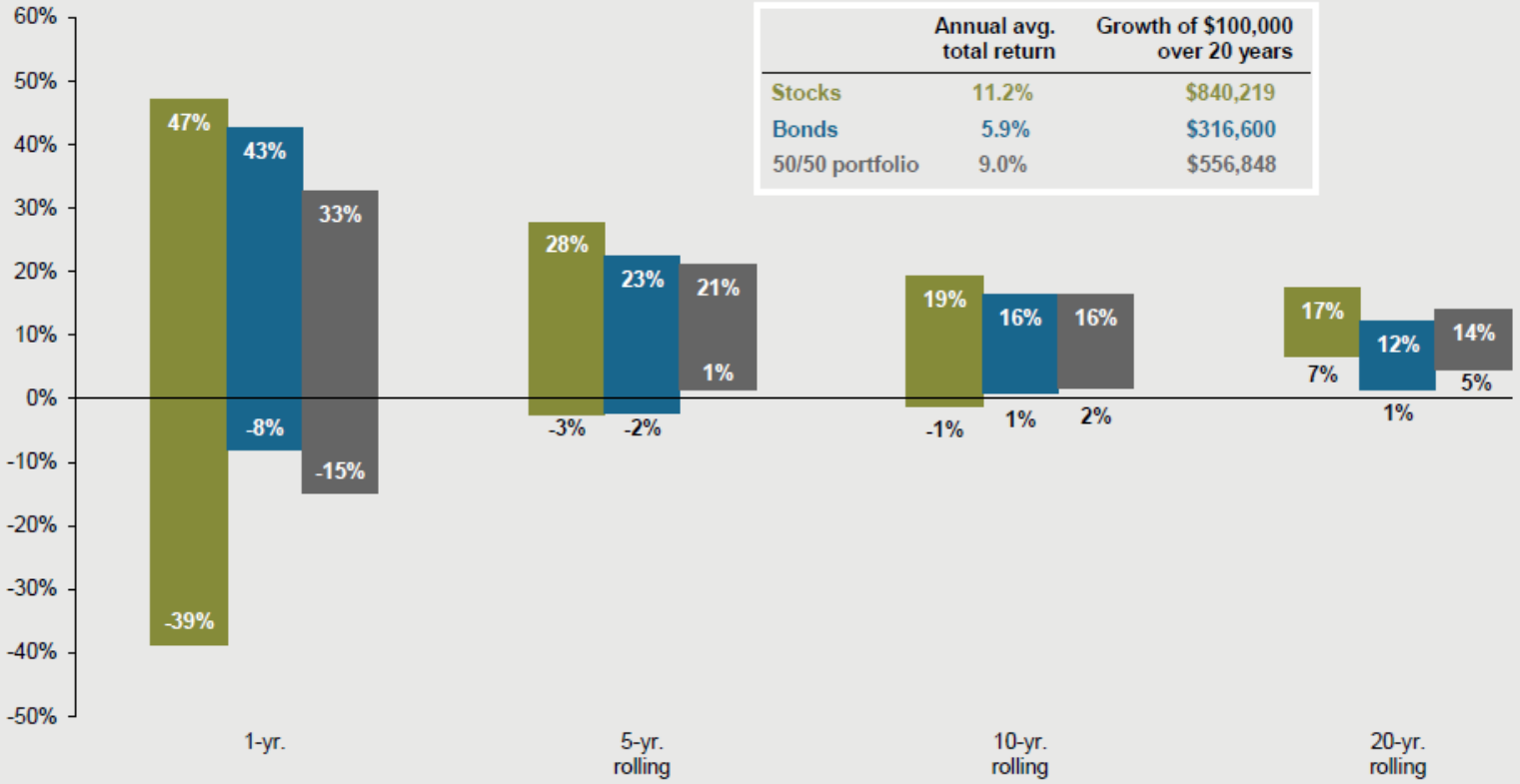
Barclays: Average Investor Emotions Can Affect Investment Returns



https://wealth.barclays.com/en_gb/home/research/research-centre/white-papers/Behavioural-Finance/Cycle-of-investor-emotions.html

STEPS TOWARD AN ACTUAL PORTFOLIO

Range of stock, bond and blended total returns
Annual total returns, 1950-2017



	Annual avg. total return	Growth of $100,000 over 20 years
Stocks	11.2%	$840,219
Bonds	5.9%	$316,600
50/50 portfolio	9.0%	$556,848

Source: Barclays, Bloomberg, FactSet, Federal Reserve, Robert Shiller, Strategas/Ibbotson, J.P. Morgan Asset Management.
Returns shown are based on calendar year returns from 1950 to 2017. Stocks represent the S&P 500 Shiller Composite and Bonds represent Strategas/Ibbotson for periods from 1950 to 2010 and Bloomberg Barclays Aggregate thereafter. Growth of $100,000 is based on annual average total returns from 1950 to 2017.
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J.P.Morgan
Asset Management

Source: (Morgan, JP 2018)

STEPS TOWARD AN ACTUAL PORTFOLIO

Correlations and volatility

	U.S. Large Cap	EAFE	EME	Bonds	Corp. HY	Munis	Currcy.	EMD	Cmdty.	REITs	Hedge funds	Private equity	Ann. Volatility
U.S. Large Cap	1.00	0.89	0.79	-0.31	0.72	-0.18	-0.51	0.58	0.66	0.83	0.87	0.85	15%
EAFE		1.00	0.90	-0.17	0.77	-0.06	-0.67	0.69	0.64	0.75	0.85	0.79	18%
EME			1.00	-0.09	0.88	0.01	-0.70	0.84	0.70	0.66	0.85	0.73	22%
Bonds				1.00	-0.04	0.83	-0.12	0.27	-0.22	0.04	-0.29	-0.39	3%
Corp. HY					1.00	0.08	-0.53	0.87	0.71	0.72	0.83	0.68	12%
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REITs										1.00	0.71	0.74	25%
Hedge funds											1.00	0.84	6%
Private equity												1.00	10%

Source: Barclays Inc., Bloomberg, Cambridge Associates, Credit Suisse/Tremont, FactSet, Federal Reserve, MSCI, NCREIF, Standard & Poor's, J.P. Morgan Asset Management.
Indices used – Large Cap: S&P 500 Index; Currencies: Federal Reserve Trade Weighted Dollar; EAFE: MSCI EAFE; EME: MSCI Emerging Markets; Bonds: Bloomberg Barclays Aggregate; Corp HY: Bloomberg Barclays Corporate High Yield; EMD: Bloomberg Barclays Emerging Market; Cmdty.: Bloomberg Commodity Index; Real Estate: NAREIT ODCE Index; Hedge Funds: CS/Tremont Hedge Fund Index; Private equity: Cambridge Associates Global Buyout & Growth Index. Private equity data are reported on a one-quarter lag. All correlation coefficients and annualized volatility are calculated based on quarterly total return data for period 9/30/08 to 9/30/18, except for Private equity, which is based on the period from 6/30/08 to 6/30/18. This chart is for illustrative purposes only.
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J.P.Morgan
Asset Management

Source: (Morgan, JP 2018)

STEPS TOWARD AN ACTUAL PORTFOLIO

Six Asset Allocation Profiles: Long Term & Short Term